## प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका Journal of Natural & Physical Sciences

#### शोध पत्रिका पटल

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## भारत के उप-राष्ट्रपति Vice-President of India

### MESSAGE

I am glad to know that the Gurukula Kangri Vishwavidyalaya, Haridwar is organising a National Seminar on "History of Mathematical Sciences and Applicable Mathematics" during March 7-9.

The Seminar offers welcome opportunity to the experts in the field of Mathematical Sciences and other related subjects to discuss the increasing relevance of Mathematics for the progress of human civilization and to evolve a strategy to face the challenges facing the discipline.

My best wishes for the success of the Seminar.

(BHAIRON SINGH SHEKHAWAT)

New Delhi 17<sup>th</sup> February 2003



## श्रीमती भावनाबेन डी. चिखालिया SMT. BHAVNABEN D. CHIKHALIA

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MINISTER OF STATE FOR
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GOVERNMENT OF INDIA
SHASTRI BHAVAN, NEW DELHI 110001



## Message

Dear Shri Singh, Pranam.

Please refer to your fax dated 20<sup>th</sup> February 2003 inviting Smt. Bhavnaben D. Chikhalia, Minister of State Tourism & Culture and Parliamentary Affairs, to inaugurate a National Seminar on "History of Mathematical Sciences and Applicable Mathematics" on 7 March, 2003. As the Parliament is in Session, Hon'ble Minister has expressed her inability to accept the invitation. However, she has asked me to convey her best wishes for the success of the Seminar.

With regards,

Yours sincerely,

Organising Secretary & Principal, Science College Gurukula Kangri Vishwavidyalaya

Hardwar 249404

Dr. S. L. Singh

Fax 01334-216366

Private Secretary

डॉ. अरूण निगवेकर अध्यक्ष



Dr. Arun Niguvekur

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D.O. No. 1-11/2003 (CM)

18th February, 2003

Dear Professor Kumar

This has reference to your letter No. 666-B of February 11, 2003. I am very happy to know that your University is organizing a National Seminar on History of Mathematical Sciences and Applicable Mathematics from March 7-9, 2003. I send my best wishes and greetings to all the participants and wish the seminar good luck and grand success.

With best wishes,

Yours sincerely,

A.T. Nin zan.

(Arun Nigavekar)

Prof. Swantantra Kumar
Patron & Vice chancellor
National Saminar on History of Mathematical Sciences and
Applicable Mathematics
Gurukula Kangri Vishwavidyalaya
Hardwar – 249 404

### सदानन्द (आई.ए.एस., रिटार्यंड) SADA NAND (I.A.S., Retd.)

Visitor Gurukula Kangri Vishwavidyalaya Former Adviser to Governor, Pubjab



## ओ३म् OM



#### गुरुकुल कांगड़ी विश्वविद्यालय हरिद्वार – 249 404 (भारत) GURUKULA KANGRI VISHWAVIDYALAYA HARDWAR -249404

118 Sector 16, Chandigarh-160015

क्रमांक/Ref. No. 03-08 VC01.

## Message

It is a matter of a great pleasure to know that the Vishwavidyalaya is organizing a National Seminar on the History of Mathematical Sciences and Applicable Mathematics from 7th to 9th March. I am sure that the seminar will highlight the achievements of ancient mathematicians of Indian and their contribution to the modern mathematics in its deliberations. The Vishwavidyalaya has been in the forefront for research activities in Vedic mathematics, and interaction in the seminar will provide a great opportunity to the scholars to interact with each other with a view to not only highlight the wisdom of ancient scholars but will also throw up new lines for the research in mathematics.

I wish the organizers and participants a great success in their venture.

Sada Nand

## सुदर्शन शर्मा कुलाधिपति SUDARSHAN SHARMA CHANCELLOR



गुरुकुल कांगड़ी विश्वविद्यालय हरिद्वार – 249 404 (भारत) GURUKULA KANGRIVISHWAVIDYALAYA HARDWAR - 249 404 (INDIA)

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दिनां क/Dated	ı						



## MESSAGE

#### Dear Prof. Swatantra Kumar:

I am pleased to know that the Mathematics and Statistics Department is organizing a National Seminar which aims at discussing development of Mathematical Science and its applicable areas. Since antiquity, Mathematics enjoys top position among all Mundane Sciences. Vedic savants are best known for making significant fundamental contribution in Mathematics and Astronomy. Modern taste of Science, Technology and Civilization has become primarily Mathematical. Our Vishwavidyalaya is well known as a centre for the study of interaction Vedic and Modern Sciences. So the theme of this Seminar is very much in tune to the Gurukuliya' tradition.

I extend a warm welcome to the peers and young delegates and wish a very successful deliberation.

(Sudarshan Sharma)

#### स्वतंत्र कुमार कुलपति SWATANTRA KUMAR VICE-CHANCELLOR





गुरुकुल कांगड़ी विश्वविद्यालय हरिद्वार – 249 404 GURUKULAKANGRIVISHWAVIDYALAYA HARDWAR-249404

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दिनां क/Dated	27.2.2003



परम हर्ष का विषय है कि गुरुकुल कांगड़ी विश्वविद्यालय के गणित विभाग में वैदिक गणित विद्या पर राष्ट्रीय विद्वत् सम्मेलन 7 मार्च 2003 से 9 मार्च 2003 तक आयोजित किया जा रहा है। जिस महा मनीषी अमर हुतात्मा स्वामी श्रद्धानन्द जी ने महर्षि दयानन्द सरस्वती के स्वप्न को साकार करने के लिए गुरुकुल की प्राक्तन शिक्षा पद्धित को संजीवन प्रदान किया उनके निर्देशानुसार ही गुरुकुल में वैदिक विद्याओं के पुनर्जागरण का कार्य निरन्तर प्रगित पर है। आज युग की पुकार है कि विश्व में अपना अस्तित्त्व बनाने तथा पूर्व मनीषियों की यशोगाथा को सत्य सिद्ध करने के लिए हमें वेदालोक में समस्त विद्याओं का उन्मेष करने पर बल देना होगा।

वैदिक परम्परा में गणित एवं ज्योतिष प्रायः पर्याय माने जाते हैं। ज्योतिष को वेदाङ्ग शास्त्रों में श्रेष्ठतम स्थान दिया गया है तथा ध्यान देने योग्य वात तो यह है कि ज्योतिष को चक्षु कहा गया है। जैसे कुछ भी देखने के लिए चक्षु का मानव जीवन में जो स्थान है वही वेद विद्या को जानने में ज्योतिष का स्थान है। ज्योतिष के आधार पर वर्ष, सम्वत्सर, मास, पक्ष, सप्ताह दिन, होरा निमेष आदि का कथन होता है। समग्र अन्तरिक्ष विद्या की जननी ज्योतिष ही है। इसी ज्योतिष वेदाङ्ग को तीन विभागों में विभक्त करके त्रिधा अंक, रेखा व बीज गणित के नाम से पुकारा जाता है। वेद में समस्त सत्य विद्याओं का बीज विद्यमान है; उस बीज को महान वृक्ष का रूप प्रदान करने में वेदाङ्ग जल सेचन का कार्य करते हैं। देश के प्राक् कालिक ऋषियों ने गहन तपश्चर्या करके वेद में विद्यमान गणित विद्या को विकसित किया है।

गुरुकुल कांगड़ी विश्वविद्यालय के मनीषी प्राध्यापक आज उसी दिशा में कार्य करते हुए इन सम्मेलनों के माध्यम से वेदों की ओर चलो का संदेश देकर वेदों की महत्ता पर प्रकाश डालेंगे। गणित विभाग के प्रोफेसर डॉ० एस०एल० सिंह, डॉ० वीरेन्द्र अरोड़ा, डॉ० वी०के० शर्मा, डॉ० महीपाल सिंह एवं डॉ० प्रभाकर प्रधान को में हृदय से साधुवाद देता हूँ कि ये सभी विद्वान् इस पावन ज्ञान यज्ञ के अनुष्ठान में पूर्ण निष्ठा के साथ लगे हैं। आज के युग में विद्वानों की, विशेषकर गुरुकुल के विद्वानों को जिस दिशा में कार्य करना चाहिए हमारे विद्वान् उसी दिशा में निःसन्देह प्रशंसनीय कार्य कर रहे हैं। उक्त सम्मेलन में विभिन्न विश्वविद्यालयों तथा शोध पीठों से जो विद्वान् इस सारस्वत यज्ञ में अपनी पावन ज्ञान की आहूति देकर सुविचार की सुगन्ध उत्पन्न करेंगे, उन सभी का हार्दिक स्वागत करते हुए सम्मेलन की पूर्ण सफलता की मङ्गल कामना करता है।

(नार्नाट्याप्र स्वतंत्र कुमार

## वेदेषु गणित विद्या बीजानि

वेद प्रकाशः शास्त्री



परमिता परमात्मना या वेदवाणी जगत् कल्याणाय मानवसृष्टेरादौ चतुर्णामृषीणां हृदयगुहायां प्रकाशिता सैव वेद वाणी सर्वज्ञानमयीति निगद्यते। सर्वाः सत्यविद्या तत एवं प्रादुर्भवन्ति। यथा विविघानां विघानां बीजानि वेदे विलोक्यन्ते तथैव गणित विद्या बीजान्यिप प्रमासमानानि सन्ति। वेदे यां छान्दसी रचना सा गणित विद्योद् गायिका। वर्णसंख्या माश्रित्य छन्दस ऊर्मयः संगीत माधुरी मुल्लासयन्ति। गणक पदं गणित विघां प्रबोध यति।

यथा -

गणानां त्वा गणपतिं हवामहे। १ ऋक् २, २३, १ गणानां त्वा गणपतिं हवामहे निधिनां त्वा निधिपतिं हवामहे। यजु० २३, १६ गण्या – १ ऋक् ३/७/५ वातं वातं गणं गणं १७ ऋक् ३/२६,६/५,५३,११

वेद संख्यावाचक शब्द प्रयोगः संख्या प्रतिपत्तये जिज्ञासून प्रबोधयति। किन्चित् समसंख्या वाचकाः शब्दाः सन्ति किन्चिच्च विषम संख्या वाचका शब्दा विलोक्यन्तो यथा विषम संख्या समुच्यते –

एका च में तिस्रश्च में तिस्रश्च में पञ्चच में पञ्चच में सप्तच में सप्तच में नवच में नवच में एकादश च में एकादश च में त्रयोदश च में त्रयोदश च में पञ्चदश च में पञ्चदश च में पञ्चदश

यजुर्वेद १८, २१६

समसंख्या वाचकाः शब्दा अपि प्राप्यन्ते – यथा – चतस्रश्च मेऽष्टौ च मेऽष्टौ च में द्वादश च में द्वादश च में षोडश च में षोडश च में विंशतिश्च में विंशतिश्च में चतुर्विंशतिश्च मे चतुर्विंशतिश्च में.....यजुर्वेद १८, २५

क्रमंशः संख्या परिगणनयामपि वेदे विलोक्यते यथा

सविता प्रथमेऽहन्थगिर्द्वितीये वायुस्तृतीये आदित्यश्चतुर्थे चन्द्रमाः पञ्चमेऋतुः षष्ठे मरुतः सप्तमे बृहस्पतिरष्टमे मित्रो नवये वरुणो दशम इन्द्र एकादशे विश्वदेवा द्वादशे।।

यजुर्वेद ३६, ६

शून्य विषये वेदेषु पर्द विद्यते तन्नामस्ति खम्। खं पदेन भवति शून्यावबोधः - यथा

ओ३म् खं बहम। यजु० ४०, १७ अंग्धि खम्। ऋक् १०, १५६, ३,

खे रथस्य। ऋक् ८, ६१, ७

कः सप्त खानि विततर्द शीर्षाणि। अथर्व १०, २, ६ विषाहि गुणते खम्।

ऋक् ४, ११, २

बीज गणित विषये सांकेतिकी दृष्टिरपि तत्रान्वेष्टुं शक्यते यथा खं पदेन शून्यं ज्ञायते बहम

आचार्य उपकुलपतिश्च, गुरुकुल कांगड़ी विश्वविद्यालयस्य

पदेन एकस्यबोधो भवति काल पदेन त्रयस्य ज्ञानं क्रियते युग पदेन चतुर्गृध्यते। एवं प्रकारेण परवर्तिमिः सूरिमिर्ननु एष संकेत क्रमः परिवर्धितः। बहुषु स्थलेषु एवं विद्याः प्रयोगा द्रष्टुं शक्यन्ते।

ज्यामिति विषये खलु तत्र त्रस्ग्वेदे स्पष्टतया प्राप्यते मन्त्रो यत्र रेखागणिताव वोधो भवति। यज्ञ वेदिमभि लक्ष्य प्रश्नमालिका या विघते सा रेखागणित प्रकाशिका।

यथा – कासीत् प्रमा प्रतिमा कि निंदानम्

आज्यं किमासीत् परिधिः क आसीत्।

छन्छः किमासीत् प्र उ गं किमुक्यं

यद् देवा देवमयजन्तविश्वे।। ऋक् १०, १३०, ३

अस्मिन् मन्त्रे प्रमा, प्रतिमा, निदानम्, परिधि, छन्द, प्र उ ग, पदानि रेखागणित बीजं व्यक्तरूपेण प्रदर्शयन्ति। एवमेव वृत्त विषये एको मन्त्रः

प्राप्यते – यथा

चतुर्मिः साकं नवतिं च नामामिः।

चक्रं न वृत्तं व्यतींखीविपत्।।

एवमेव वर्ष चक्र वर्णन माध्यमेन प्राप्ताः

शब्दा सन्ति – यथा –

द्वादश प्रघयश्चक्रमेकं त्रीणि क उ तच्चिकत।

तस्मिन् साकं त्रिशता न शंकवो

अर्पिता षष्टिर्न चलाचलास:।।

ऋक् ११६, १७ १७८

विदुषां कर्त्तव्यमिदं विघते सम्प्रति यत्ते वेदे निस्तन्द्रा मूत्वा निगूढं तत्वजातं सात्विकिधया चिन्तयन्तु। हर्षस्यायं विषयो विघते गुरुकुल कांगड़ी विश्वविद्यालये गणित विभागे राष्ट्रीयं विद्वत् सम्मेलनं वैदिक परिप्रेक्ष्ये समायोजकैरायोज्यते। एतदर्थं प्रो एस.एल. सिंह प्रो. वीरेन्द्र अरोड़ा प्रो0 वी0के0 शर्मा प्रो. मृहीपाल, डाॅ० पी. प्रधान पदवाच्याः सर्वे घन्यवादार्हाः सन्ति। सम्मेलनस्य सार्वित्रकं गौरवं कामये।

ओ३म् *डॉ० महावीर अग्रवाल ॐ<sup>१९५१</sup>०*४

प्रोफेसर संस्कत एवं कलसचिव

गुरुकुल कांगड़ी विश्वविद्यालय Gurukula Kangri Vishwavidyalaya पोस्ट गुरुकुल कांगड़ी, हरिद्वार – 249404 P.O. Gurukula Kangri, Hardwar-249404



## संदेश

## बृहत्संहिता में वास्तु विज्ञान

गुरुकुल कांगड़ी विश्वविद्यालय का यह सौभाग्य है कि गणित विभाग द्वारा आयोजित राष्ट्रीय शोध संगोष्ठी में देश के मूर्धन्य विद्वान् उपस्थित होने जा रहे हैं। एक मास से भी न्यून अविध में गणित के लब्ध प्रतिष्ठ विद्वान् प्रो० एस०एल० सिंह, प्राचार्य विज्ञान महाविद्यालय तथा प्रो० वीरेन्द्र अरोड़ा, संकायाध्यक्ष ने देश के वयोवृद्ध, ज्ञानवृद्ध विद्वानों को इस आयोजन में निमन्त्रित किया है और ये वरेण्य विद्वान् यहां उपस्थित हैं।

गणित को अनेक विद्याओं का मूल कहा जाता है। संसार का कोई व्यक्ति गणित विद्या से अछूता नहीं है। विज्ञान्, कप्म्यूटर, अभियान्त्रिकी, प्रौद्योगिकी, वाणिज्य, अर्थशास्त्र आदि का तो यह प्राण है। संपूर्ण संसार को गणित विद्या की शिक्षा देने वाले भारतीय आचार्यों ने ही शून्य की गवेषणा की थी। ज्योतिष वेदांग के अंग रूप में इस विद्या का विस्तार यहां होता रहा है। मैं शोध संगोष्ठी की पूर्ण सफलता की मंगल कामना करते हुए तथा विशेष रूप से प्रो० एस०एल० सिंह जी के प्रति हार्दिक सम्मान प्रकट करते हुए वास्तुशास्त्र पर कुछ विचार बृहदसंहिता के आधार पर प्रस्तुत कर रहा हूँ।

भवन निर्माण कला सौन्दर्य से जुड़ा हुआ विज्ञान एवं कला दोनों है, जिसका काम व्यवस्थित एवं सुसज्जित भवनों के माध्यम से भावनाओं को व्यक्त एवं जागृत करना है। समराङ्गण सूत्रधार के रचयिता भोजने स्थापत्य को इस प्रकार परिभाषित किया है।

### शास्त्रं - कर्म तथा प्रज्ञां शीलं च क्रिययान्वितम्।

वास्तु शब्द 'वस् धातु' से निर्मित है जिसका अर्थ यास्क के निरूक्त में "रहना" है। पुरातत्वाविदों ने सिन्धु घाटी सभ्यता के समय के बहुत से स्मारक एवं नगर नियोजन उत्खनन में पाये हैं। ऋग्वेद एवं अन्य वैदिक ग्रंथों में भी भवन निर्माण से जुड़े हुए महत्वपूर्ण शब्द मिलते है, यथा – वास्तु, वास्तोष्पति, अर्म, अर्मक, गय, हर्म्य, पस्त्य, ओकः, गृहं, दम आदि। संस्कृत में ऐसे अनेक ग्रंथ हैं जोिक भवन निर्माण, नगर नियोजन, सहायक अभियांत्रिकी एवं शिल्प के मुख्य आयामों को बताते हैं। उनमें से मुख्य है – अग्नि मत्स्य संहिता, कलिकागम, गर्गेसंहिता, नारद संहिता, प्रसाद मंडन वास्तुशास्त्र, मानसार, समरांगणसूत्रधार, विश्वकर्मा प्रकाश आदि।

इन ग्रन्थों में वराह मिहिर की बृहत् संहिता ज्योतिष, खगोल शास्त्र, मणि विज्ञान, धातु विज्ञान, कृषि भवन निर्माण कला, अर्थशास्त्र का विश्वज्ञान कोष है। वराह मिहिर (505 ई0) के सम्बंध में कहा जाता है कि ये उज्जैन के निकट रहने वाले एक विशेष प्रतिभा के धनी वैज्ञानिक

थे जो कि प्राचीन परम्पराओं एवं ऋषि मुनियों में विश्वास रखते हुए विषय के अध्ययन में वैज्ञानिक पद्धति अपनाते थे। उत्पल के अनुसार वराह की निम्न रचनायें हैं। (1) पंचसिद्धांतिका (2) जातक (3) यक्षेस्व – मेधीय यात्रा (4) योगमाला (5) विवाह पतल (6) बृहद् संहिता। आगम शास्त्र एक विशेष विधा है जिसमें भवन निर्माण, सामुद्रिकी, आयुर्वेद, रत्न विज्ञान आदि पर विद्वत्ता पूर्ण चर्चा की गई है। वराह ने अध्याय 53 में भवन निर्माण के कुछ अत्यन्त महत्वपूर्ण बिन्दुओं पर प्रकाश डाला है। जोकि इस प्रकार हैं –

(1) भवन निर्माण के लिए भूमि का चयन – किसी भी प्रकार के भवन के लिए भूमि चयन की विशेष महत्ता है। वराह ने कुछ ऐसे तथ्यों का वर्णन किया है जो कि स्थायी निर्माण के लिए सहयोगी हो सकते हैं जिसके अनुसार भूमि कोमल, सुगन्धित व मधुर स्वाद वाली होनी चाहिए। नीचे से खोखली नहीं होनी चाहिए और लाभवायक औषधियों वनस्पतियों से भरी होनी चाहिए।

शस्त्रौषधिद्रुमलता मधुरा सुगन्धा। स्निग्धा, समा न सुषिरा च मही नराणाम्।। (खृ. सं, ८८)

मिट्टी जो कि शुभ्र, लाल पीली, और काली होती है, विभिन्न वर्ग के लोगों के लिए लाभदायक होती है। किन्तु आवश्यक यह है कि यह विभिन्न प्रकार की घासों से ढकी होनी चाहिए – दर्भा, सार, दूब और कांस। लेकिन मुख्य सिद्धांत है आत्म सन्तुष्टि –

तत्तस्य भवति शुभदं यस्य च यस्मिन् मनो रमते। ब्रु.सं. 64

वही स्थान शुभ है जहाँ मन प्रसन्नता अनुभव करे। संक्षेप में भवन बनाने के लिए वहीं भूमि उपयुक्त है जो कि समतल हो, उपजाऊ और कोमल हो, जिससे कि भवन बिना बाधा के बनाया जा सके।

(2) विभिन्न प्रकार के निवास करने वाले घरों की विभाए – वराह ने राजा या धनी व्यक्तियों के लिए तथा दैवज्ञ पुरोहित, वैद्य आदि विभिन्न प्रकार के नागरिकों के लिए घरों की माप का वर्णन किया है –

उत्तममष्टाभ्यधिकं हस्तशतं नृपगृहं पृथुत्वेन। अष्टाष्टोनान्येव पञ्च सपावानि दैर्ध्येण।। बु.सं. 54

राजा का महल पाँचों में से एक विशेष प्रकार का होता है उनमें सबसे उत्तम चौड़ाई में होते हैं और अन्यों में 50, 46, 42, 54 मीटर और 38 मीटर क्रमशः होते हैं। उनकी लम्बाई चौड़ाई से एक चौथाई अधिक होती है।

इसी प्रकार सेनापित, युवराज महामंत्री और दूसरे अधिकारियों के घरों के माप का वर्णन 5 से 9 वें पवों में मिलता है। यहाँ आगे कहा गया है कि दैवज्ञ, पुरोहित और वैद्य के गृहों में चौड़ाई में 20, 18 मीटर और 16 मीटर, उनकी चौड़ाई से 1/6 वाँ हिस्सा अधिक होनी चाहिए।

वराह के अनुसार सामान्यतः एक भवन की लम्बाई और चौड़ाई नाप में बराबर होनी चाहिए। केवल उन घरों को छोड़कर जहाँ पर एक बड़ा हाल होता है। वहाँ लम्बाई, चौड़ाई से दो गुनी होनी चाहिए।

वास्तुनि यो विस्तारः स एव चौच्छ्राय निश्चयः शुभवः। शालैकेषु गृहेष्वपि विस्तारादद्विगुणित-वैघूर्यम्।। स्नु.सं.।।

पशुओं के लिए शस्त्रागार के लिए यज्ञशाला एवं खेल आदि के लिए कोई विशेष माप नहीं दी गई है। वराह ने हालांकि यह अनुभव किया कि विद्वान् वास्तु शास्त्रों के अनुसार रहने वाले स्थानों की लम्बाई 50 मीटर से अधिक नहीं होनी चाहिए।

बु.सं. 16

विस्तारषोडशांशः सचतुर्हस्तो भवेद गृहोच्छायः।

हादशभागेनोनो भूमौ भूमौ समस्तानाम्।। बृ.सं. 22

प्रथम तल की ऊंचाई 1/16 वाँ हिस्सा चौड़ाई से कम होनी चाहिए। उससे ऊपर के तलों की ऊंचाई निचले तल से 1/12 वाँ भाग कम होनी चाहिए।

वराष्ठ के अनुसार घर की वीवारें पकी हुई ईंटों की तथा कमरों की चौड़ाई के 1/16 वाँ भाग होनी चाहिए। लेकिन लकड़ी से बने हुए घरों पर कोई प्रतिबन्ध नहीं था।

व्यासात् षोडशभागः सर्वेषां सद्मनां भवति भित्तिः।

पक्वेष्टकाकृतानां दासकृतानां तु न विकल्पः।। बृ.सं. 23

बरवाजों, खम्भों आदि की नाप भी यहाँ दी हुई है। (24-36) राजा अथवा धनी और किसी विद्वान् का घर जिसके चारों ओर बरामदा होता है और चार दरवाजे होते हैं 'सर्वतोभद्र' कहलाता है।

> अप्रतिषिद्धालिन्दं समन्ततो वास्तु सर्वतोभद्रम्। नृपविबुधसमूहानां कार्यं द्वारैश्चतुर्भिरपि।। बृ.सं. ३१

वराह ने निवास स्थानों का नामकरण दीर्घप्रकोष्ठों और कक्षों की स्थिति के आधार पर किया है। तीन कक्षों के घर को जिसमें उत्तरी प्रकोष्ठ नहीं है को 'हिरण्यनाभ' कहा गया है। इसी प्रकार 'सुक्षेत्र' में पूर्वी प्रकोष्ठ नहीं होता है। दक्षिण प्रकोष्ठ से रहित घर को कली और पश्चिम हाल से रहित घर को पाकसघना कहते हैं।

दो दीर्घ प्रकोष्ठों घर जिसमें केवल पश्चिमी और दक्षिणी प्रकोष्ठ होते हैं, 'सिद्धार्थ' के नाम से जाना जाता है। जिसमें पश्चिमी और उत्तरी प्रकोष्ठ होते हैं, यमसूर्या; जिसमें उत्तरी और पूर्वी प्रकोष्ठ होते हैं, वट; जिसमें पूर्वी और पश्चिमी प्रकोष्ठ होते हैं, गृहचुल्ली और जिसमें दक्षिणी और उत्तरी प्रकोष्ठ होते हैं उसे 'काच' कहते हैं।

51 – 69 मंत्रों में वास्तुपुरूष एवं उसके सहयोगी देवताओं का वर्णन है। वराह ने रसोई भण्डार घर आदि की दिशाओं का भी वर्णन किया है।

> ऐशान्यां देवगृहं महानसं चापि कार्यमाग्नेय्याम्। नैर्ऋत्यां भाण्डोपस्कमरोऽर्थधान्यानि मारुत्याम्।। बु.सं. ११८

घर में पूजा स्थल उत्तर पूर्व में, रसोईघर दक्षिण पूर्व में, भण्डार घर दक्षिण पश्चिम में और कोषागार एवं अन्नागार उत्तर पश्चिम में होना चाहिए।

वराह के अनुसार – विशेष प्रकार के वृक्ष घर कें चारों ओर लगाए जाने चाहिए। घर के चारों ओर कांटेदार वृक्ष जैसे खिदर, दूधी – वृक्ष जैसे अर्क और फलदार खड़े वृक्ष जैसे कि आम आदि नहीं लगाने चाहिए। इसका कारण यह है कि इनसे दुर्घटना हो सकती है। समीप रहने वाले लोगों को भय हो सकता है। ये वृक्ष भवनों को हानि भी पहुँचा सकते हैं।

वराह ने परामर्श दिया है कि यदि उपरोक्त वृक्ष घर के आसपास है तो उन्हें उखाड़ना नहीं चाहिए अपितु कुछ सुंदर व उपयोगी वृक्ष उनके मध्य में रोपित कर देने चाहिए। ऐसे अच्छे वृक्षों में निम्ब (नीम), वक्ल, अशोक आदि है। ऐसे वृक्ष भवन की सुंदरता को तो बढ़ाते ही है साथ ही मानव जीवन को भी स्वस्थ व सुखी रखने में सहयोग प्रदान करते हैं। ऐसा विचार आज भी पर्यावरण की दृष्टि से अत्यंत महत्वपूर्ण है। अगला प्रश्न है कि हमारा पड़ोसी कौन हो? सचिव और धूर्त व्यक्ति बिल्कुल नहीं। ऐसे लोग धन छीन सधकते है या बच्चों के जीवन को भय उत्पन्न कर सकते है। घर चैत्य के निकट नहीं होना चाहिए। जिसके अनेक अर्थ हैं – पत्थरों का ढेर, श्मशान भूमि, बौद्ध या जैन मंदिर या गूलर का वृक्ष। इसी प्रकार घर के चारों ओर वाल्मीक (बांबी) और गढ्डा नहीं होना चाहिए क्योंकि ये निवासियों के जीवन के लिए खतरा बन सकते है।

बहुत से कारण हैं जोिक निवास स्थान की सुंदरता और पवित्रता में सहयोग प्रदान करते हैं। वराह के अनुसार – मुख्यद्वार प्रत्यक्ष रूप से विशाल और घर के अन्य सभी द्वारों से अलग होना चाहिए। मुख्यद्वार किसी भी दूसरे द्वार से अलग होना चाहिए तथा किसी भी अन्य द्वार से निर्माण एवं सुन्दरता में छोटा नहीं होना चाहिए। यह द्वार पवित्र वस्तुओं जैसे घड़ा, फलों का गुच्छा आदि से परिपूर्ण होना चाहिए। इस प्रकार के भव्य मुख्य द्वार से जो भी व्यक्ति अंदर आता है वह प्रवेश करते समय प्रसन्नता अनुभव करता है।

वराह के अनुसार पूरा घर विभिन्न प्रकार के फूलों, सुंदर आर्क (अर्धवृत्त) सुसिज्जित जलपरिपूर्ण कलशों से शोभायमान होना चाहिए। घर को विभिन्न इत्रों से सुगन्धित रखना चाहिए। घर को पवित्र वैदिक मंत्रों के पाठ से धार्मिक बनाकर रखना चाहिए।

भूरिपुष्पविकरं सतोरणं तोयपूर्ण किणशोपशोभितम्। धूपगन्ध खलिपूजितामरं ब्राह्मणध्वनियुतं विशेदगृहम्।। (ब्रु.सं. 25)

उपर्युक्त कारक न केवल घर को सुशोभित करते है अपितु उस घर में रहने वालों की प्रसन्नता और विश्राम में भी सहयोग करते है। आज भी आंतरिक सज्जा भवन निर्माण की एक प्रमुख शाखा है।

इस विषय में वराह के परामर्श व्यावसायिकों के लिए बहुत उपयोगी है, विशेषतः गृहणियों के लिए सदैव उपयोगी रहेंगे।

इस प्रकार वराह भवन निर्माण कला के विभिन्न पक्षों को प्रकट करते हुए यह बताते हैं कि भारत ने 1500 वर्ष पूर्व किस प्रकार सामाजिक एवं सांस्कृतिक प्रगति की थी? सभ्यता का उच्च स्तर एवं शाही रहन – सहन इस ग्रंथ में झलकता है।

वराह अपनी स्पष्ट, पारवर्शी एवं आनन्दबायक भाषा शैली से भवन निर्माण विज्ञान के विभिन्न पहलुओं का वर्णन करने में सफल हुए।

भवन निर्माण के साथ सही प्रकार के पौधों के रोपण को भी महत्व दिया गया। यह परामर्श आज भी उपयोगी है जब हम वनों के कटान और पर्यावरण असंतुलन की बात करते हैं। वस्तुतः भवन निर्माण का अध्ययन ज्ञानवर्धक एवं प्रेरणादायक दोनों है।

### मदन कौक्षिक MADAN KAUSHIK सदस्य विधान सभा, उत्तरांचल Member of Legislative Assembly, Uttaranchal



कार्यालय : 4 - ए, हरिनगर ज्वालापुर

हरिद्वार – 249407

निवास : 224 खी, गली नं0 7 खन्ना नगर

ज्वालापुर (हरिद्वार)

फोन : 265757(का0) 25803 (नि0)

दिनांक 26-02-2003



## Message

This is a matter of great pleasure that the National Seminar on History of Mathematical Sciences and Applicable Mathematics is being organized by the Department of Mathematics and Statistics, Gurukula Kangri University, Hardwar form 7th to 9th March, 2003. There should be a good interaction and exchange of ideas between mathematicians and traditional scholars not only to extend the frontiers of mathematics but also to discover ways and means of harnessing the forces of knowledge for the service of human race and for betterment of quality of human life.

I extend my best wishes for the success of the Seminar.

Madan Kaushik

### FROM ORGANIZING SECRETARY - S. L. Singh

This is an overwhelming occasion that we are organising a National Seminar on History of Mathematical Sciences and Applicable Mathematics. We could get only 28 days to organize this Seminar. First I contacted Professor G. S. Panday of Ujjian and Professor B. S. Yadav of Dehli, and they very kindly not only agreed to deliver lectures but also extended their full cooperation for organizing this Seminar at a very short notice. Our Vice-Chancellor Professor Swatantra Kumar has shown special interest and enthusiasm in this Seminar. Indeed, we are able to organize this seminar because of his dynamic leadership and valuable co-operation from the administration, viz, Professor Ved Prakash, Pro-VC, Professor Mahavir Agrawal, Registrar and Shri Jai Singh Gupta, Finance Officer.

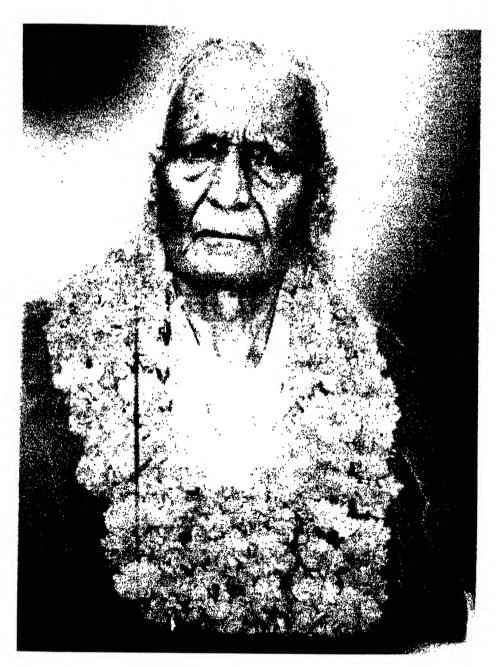
On behalf of the organizing committee and my own behalf, I extend a very warm welcome to our Chief guest, distinguished guests, & scholars, delegates, friends, colleagues, brothers and sisters.

We organized national level symposia on Mathematics and Vedic Mathematics in 1989 & 1995; the 64th Indian Mathematical Society Conference and Symposia on Fixed Point Theory & Vedic Mathematics were also organized in 1998. This was followed by an International Conferences on History of Mathematics in 1999 which was organized under the chairmanship of Professor B.S. Yadav. All these events were very well received by scholars and Mathematicians. Perhaps, this is the last seminar that I am organizing with very active cooperation of my colleagues, with the risk of missing many names; I must mention Virendra Arora, V. K. Sharma, M. P. Singh, Prabhakar Pradhan, D. D. Sharma, R. D. Kaushik, Rajendra Kumar Agrawal, P. P. Pathak, L. P. Purohit, S. C. Dhamija, R. D. Singh, Shri Krishna and Karmjeet Bhatia. Once again, I take the risk of missing a few more names of my students and non teaching staff and mention the untiring

work of Ajendra Kumar, Anurag Naithani, Ashish Kumar, Hansraj Joshi, Yashpal Rana, M.G. Upadhyaya and Prakash Tiwari.

I take this opportunity to extend my indebtedeness and sincere thanks to Dr. Raj Kumar Rawat (G. K. Pharmacy), Mr. K. L. Kashyap (Mandi Govind Garh Charitable Trust Dharmshala, Hardwar), Mr. Raj Kumar Mahajan (Mahajan Bhawan) and Mr. T. P. Kumaria (GKV's consultant architect), Mr. Rajendra Tiwari (basically Mathematician and Asst. Transport Commissioner, Uttaranchal Pradesh), his colleague Mr. M. M. Jaiswal, and Mr. Chandra Kiran Saini. However, cooperation alone can not bring happiness unless there is a good support involving finances. I take this opportunity to express indebtedeness to His Holiness Swami Chidanandji (Chief President, Parmath Niketan, Rishikesh), Rajrani Trust (Jalandhar), Shri Madan Kaushik (M.L.A., Hardwar), Shri M. C. Garg (Periodica India, Faridabad), M/s Oriental Assurance Co. (Hardwar), Trustees of Mandi Govind Garh Charitable Trust Dharmashala, Hardwar, General Manager, Gurukula Kangri Pharmacy, Hardwar and Furniture House (Hardwar).

Finally, I extend a very warm welcome to all friends and delegates, and hope that their active participation alone can bring a great success to the Seminar.



Harbanshlal Sharma (02.02.1920 - 02.12.2002)

## दानवी२ - ह२बंश लाल शर्मा

#### वीरेन्द्र अरोडा

गुरुकुल कांगड़ी विश्वविद्यालय के पूर्व कुलाधिपति, आर्य प्रतिनिधि सभा पंजाब के प्रधान पंठ हरवंशलाल जी का जन्म 2 फरवरी, 1920 में जालन्धर जनपद के रूड़की कला ग्राम में एक पौराणिक ब्राह्मण परिवार में हुआ था। आपके पिता श्री कर्मचन्द शर्मा और माता श्रीमती लालदेवी बहुत ही धार्मिक विचारों के थे। करांची में रहते हुए ही पंठ कर्मचन्द जी अपने भाईयों के साथ आर्य समाज से जुड़ गये थे। सभी भाई आर्य समाज की विचारधारा में ऐसे रंग गये कि आर्य समाज के दीवाने हो गये। माता-पिता का प्रभाव बालक हरवंशलाल पर बहुत गहरा पड़ा। ज्येष्ठ पुत्र होने के नाते इन्होंने आर्य समाज तथा साधु-विद्वानों की सेवा का कार्य बाल्यावस्था से ही प्रारम्भ कर दिया।

कुछ वर्ष करांची में नौकरी के पश्चात अपने एयरफोर्स में 10 वर्ष तक सेवा की और फिर नौकरी को तिलांजली देकर साईकिल पार्ट्स का कार्य प्रारम्भ कर दिया। ईमानदारी, परिश्रम, लगन एवं ईश्वर विश्वास आदि गुणों से आपका उद्योग दिन-दूनी रात चौगुनी प्रगति करता रहा। इनकी धर्मपत्नी श्रीमती राजकुमारी भी बहुत धार्मिक विचारों की हैं।

2 दिसम्बर 2002 को अपनी जीवन यात्रा पूर्ण कर पंडित जी अनन्त में विलीन हो गये। पंजाब के इस भामाशाह की मृत्यु से जहां कई सामाजिक, धार्मिक एवं शैक्षणिक संस्थायें अनाथ महसूस करने लगीं, वहीं आर्य जगत ने भी एक अमूल्य हीरा खो दिया।

मैं पंडित हरवंश लाल शर्मा जी से सन् 1965 से परिचित हूँ, जब मैंने गुरुकुल में शिक्षण कार्य प्रारम्भ किया था। मुझे उनके निकट आने का भी सौभाग्य प्राप्त हुआ। पंडित जी एक सरल, सच्चे आर्य-समाजी, धार्मिक एवं दयावान व्यक्ति थे। वे सदैव नम्र रहते थे। पंडिर जी कर्मठ, कर्तव्य परायण निष्ठावान व दानवीर व्यक्ति थे। गुरुकुल के प्रति उन्हें विशेष प्रैम था। वे गुरुकुल के लिये प्रत्येक प्रकार की सेवा के लिये तत्पर रहते थे।

गीता में श्री कृष्ण ने यज्ञ, दान और तप को नित्य करणीय कर्म बताया है। यज्ञ से देवताओं की तृष्ति होती है। नियत समय पर वर्षा और अन्न उत्पन्न होते हैं। दान से प्राणिमात्र के जीवन की रक्षा होती है तथा तप से व्यक्ति संयमी, परोपकारी और सेवा भावीं

हो जाता है। व्यक्ति को समाज और राष्ट्र से जोड़ने में यज्ञ, दान और तप की मुख्य भूमिका बताई गई है। पंडित हरबंश लाल शर्मा जी में तीनों गुण विद्यमान थे। उन्हें धनी होने का लेश मात्र भी अभिमान नहीं था। नित्य यज्ञ और दान करना उनके जीवन के अभिन्न अंग बन गये थे। आर्य समाज की विभिन्न संस्थाओं को इस कार्य के लिये वह पर्याप्त राशि सात्त्विक भाव से प्रदान करते थे। उनके हृदय में कभी प्रतिदान की भावना पैदा नहीं हुई। उन्होंने निष्काम भाव से पुण्य-कार्य किए। इसलिए उनके दान को सात्त्विक कहा जाना चाहिए। अपने जीवन में उन्होंने इसी भाव से प्रत्येक व्यक्ति की सहायता की।

गुरूकुल कांगड़ी विश्वविद्यालय और स्वामी श्रद्धानन्द उनके आदर्श रहे हैं। उनकी मान-मर्यादा के लिए पंडित जी सदैव तत्पर रहते थे। वह निर्भीक, निर्लोभ तथा आत्म-विश्वास से पूर्ण व्यक्तिव के धनी थे। वह अत्यन्त विनम्रता के साथ अपनी बात कह देते थे पर उनका दृष्टिकोण कठोर और हितकारी होता था। पंडित जी ने सदैव धर्म तथा नीति की बात कही तथा सच्चे मन से जो कहा वही किया। गुरूकुल की प्रबन्ध नीति में उनकी यही दृष्टि थी।

पंडित जी के आदर्श एवं भावालोक में उनके द्वारा सत्यापित ''राजरानी ट्रस्ट, जालंधर'' ने वर्तमान सेमीनार के आयोजन हेतु अपना सराहनीय सहयोग प्रदान किया। एतदर्थ आयोजन समिति आभार मानती है।

#### THE GURUKULA KANGRI PHARMACY

#### R. K. Rawat\*

The Gurukula Kangri Pharmacy is preparing ayurvedic medicines of best quality from last one century. Our products created their own place in the hearts of public due to best standard in quality and high ideality in preparation as mentioned in old Ayurvedic books. Now a days, we are one of the largest producer of ayurvedic drugs in India.

We are preparing almost 400 products. We are also developing new products related to various diseases. The potency of our medicines is far better and it is continuously increasing by modern techniques. The main branches of medicines, which we are preparing are Powders, Bhasm, Ras & Vati, Pak-Avaleh, Asava-Arishta, medicated oils and Ghritas.

Many new products are under consideration for preparation and testing. From the above mentioned products, the important products, which have leading sale are Bhimseni Surma, Chyavanprash, Payokil, Shilajeet, Madhumeh Nashini Gutika, Gurukul Chai & Gold preparations like Vrihat Vat Chintamani, Basant Kusumakar Ras & Yogendra Ras.

Since the unit is situated in the foothills of Himalaya, we are much capable to arrange fresh medicinal plants in comparison to other ayurvedic drugs producing units. The identification of purchased medicinal plants (crude drugs) is done by the expert ayurvedic doctors and chemists. After their approval, plants or medicinal material are used for preparation. This factor maintains the quality of products in one way.

The other way to improve quality is the methods of preparation of drugs. We are somewhat different with other ayurvedic drugs producing units because we are not compromising with the old methods of preparation of drugs given in standard books of Ayurveda. For this purpose, the strict supervision is done by the Aurvedic doctors.

The washing of crude material, crushing and grinding of materials or products is

<sup>\*</sup>General Manager, Gurukula Kangri Pharmacy, Hardwar.

done by the help of machines to make work faster and for other preparative methods, we are strict to traditional Ayurvedic standards.

Our products, specially Asavs and Arishtas are prepared in four to six months by self generating alcohol. So their potency against diseases has been recorded very high. Indeed, from all points, we proudly can say that due to high quality, our products are highly potent.

The packaging of medicines is done in different sizes of sealed packs. The packaging label has been printed by the weight of medicine, cost, year and batch number. Before purchasing medicine, it should be ensured that whether the seal is properly intact or not.

The drugs are sold through various agencies all over India.

The profit earned by the pharmacy is used for charity. The profit is donated for the education of students at Gurukula Kangri Vidyalaya (boys), Haridwar and Kanya Gurukula (girls), Dehradun.

### **MATHEMATICS AND VEDIC LITERATURE**

#### VIRENDRA ARORA

This paper intends to present a bird veiw of mathematical incidences in Vedic and post-Vedic literature; refer to Astrotree (see the inside cover).

It is a well established fact that Geometry of Aryans is predominated by arithmetic. Actually 'Hindus' algebraised geometry and the geometry of Indians have been developed out of necessity for the construction of Various *agnicitus*, *vedis*, *mandaps* etc. which are required for sacrificial ritual.

Present discussions are confined only to some of incidences of geometrical aspects related to Vedas and *Brāhamanas*.

The greatness of scientific discoveries made by ancient Indian seers is being progressively realized and admired by the modern scientific world. One of the scientific luminaries who has brought credit to this country for his outstanding discoveries and achievements in astronomy is the great Āryabhaṭa; generally known as the father of ancient Indian Mathematics. One of the great discoveries of this Indian astronomer of the 5th Century A.D. is that the stars are fixed and it is the earth which moves creating the phenomenon of the daily rising and setting of the Sun and the other planets and constellations by its motion on its axis as well as around the Sun.

भवंजरः स्थिरो भूत्वा वृत्यावृत्य प्रतिदैवसिकौ उदयास्तमयौ संपादपति ग्रह नक्षत्राणाम्।। (Arya Siddhanta)

According to social reformer Svāmī Dayānand Sarasvati: All science and technology that is or will ever be is contained in the seed form in the Vedas.

The term 'Veda' literally means 'Knowledge, All-Knowledge, 'Gnosis'. It is all reasoning and analysis. वेद विद (Veda Vid) means one who knows and acts up to the Vedas. It ultimately makes the man up righteous and such a man is ever beneficial to the people of the world at large.

Vedas are the words of God, revealed in the beginning of creation for the moral, spiritual and physical guidance and uplifts of humanity. They are replete with eternal truths and throw a light on various aspects of life to make a man perfect and ideal.

God, out of his infinite source of knowledge reveals in the beginning of creation a part of it adequate for the requirements of the soul, its spiritual satisfactions, fulfilment of its thirst for truth and making its journey of life successful.

God is infinite, the soul is finite, therefore soul can not contain the infinite knowledge of God. God creates, sustains and dissolves the world.

Whenever the world is created God reveals the Vedas. This process is going on since eternity and will go on for ever.

There are 20380 verses in all the four Vedas namely Rg, Yajura, Sāma and Atharva as follows:

Rg Veda	10522		
Yajura Veda	2006		
Sām Veda	1875		
Atharva Veda	5977		

For correct interpretation of the Vedas, the words should be taken in their analytical sense, i.e., root-meaning should be taken into account of Vedic words. The Veda only the सहिता (the real text of the Vedas) as revealed by God in the beginning of Creation on Agni, Vāyu, Āditya and Angiras, the four Rshis competent to receive God's Revelation.

Mathematics is the base of science. It is in the root of creation of the universe. There exists a motion in every particle of the universe which is related to mathematics; Times of sunrise, sunset, eclipses etc. are decided on the basis of calculations.

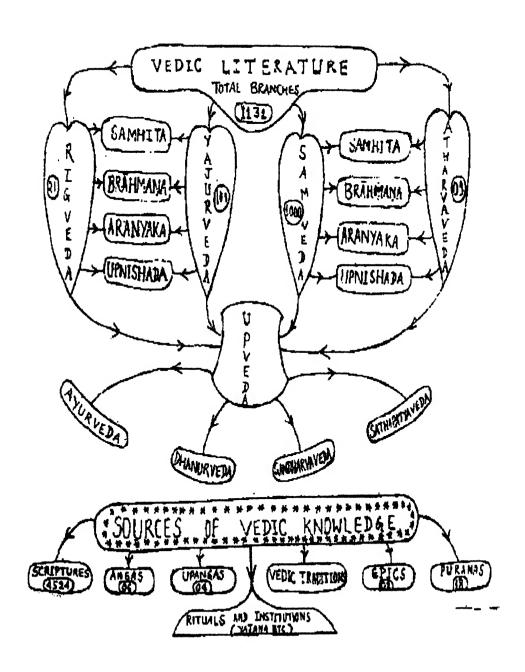
Also Mathematics has the highest place in Vedang-astrology.

यथा शिखा मयूराणां नागानां मण यो यथा। तद् वद् वेदागं शास्त्राणाां गणितं मुर्घनि स्थितम्।। वेदागं ज्योतिष (याजुष)4

For accurate calculation of times of certain festivals and of times auspicious for acts of worship, good knowledge of arithmetic, plane, spherical trigonometry and geometry is required.

In Vedic religions, for purpose of worship, different types of altars are required. These altars have definite shape and area. They can be in the shape of square, triangle, semicircular according to worships for different purposes.

These acts of worships are found in the remotest towns. Several references are available in Rg Veda Samhita. The science of the contributions of altars take a mere specific



form in the *Taitrireeya Samhita* and *Taitrireeya Brāhamana*. It may be appropriate to mention that there was provision of these altars in the hermitage of sage Agasthya, and in Rama's own hermitages both at Chitrakoot and Panchavati.

For accurate construction of these altars, the unit of length used was *Vyam* of *Vyayam* (व्याम) which was about 96 inches. Possibly this was the average height of the average man in those days, that is why *Puruṣha* (पुरूष) was also used for this unit of length. The area of the altars used to be exactly one square *vyam*.

According to Hindu traditions of the hereditary handing down of instructions from father to son, and from the Guru to the disciple, the need for setting out instructions in written form was felt. Therefore several *sulba sūtras* were written, which were known as the "Śrauta Sūtras". The meaning of the word sulv is to measure, later on it was also known as Rajju (rope). The name Rekhā Ganita is of later origin.

Out of seven śul'bas known at present, Baudhyāna, Āpastamba, Mānava, Maitrayaṇa, Vāraha and Vādhula belong to Ķṛṣṇa Yajurveda. These śulbas are after the name of Rishi who wrote them. Baudhāyana, Apastamba and Kātyāyana śulvas are of importance from mathematical point of view. The date of these Śulvas-Sūtra are estimated to be between 800 B.C. and 500 B.C.

It is important to mention that the writers of these Śulba's only wrote them and codified the rules for the estimations of the altars, which were in existence from ancient times. They did not specify and clarified the rules for the estimation of altars.

The *Sulvas* explain a large no. of simple geometrical combinations - estimation of square, rectangles, parallelogram which involves no. of theorems namely:

The diagonal of rectangle divide it into two equal parts and so on, however how these theorems are actually obtained, there in no definite answer. It may be concluded that these theorems were asserted as matter experience and measurement. The *sulvas* are not formal mathematical treatises, they are only adjuncts to certain religious work.

The above remarks do not conclude that there was no geometry before the existence of *śulbas*. Geometry was very well known before *śulva's* period.

Geometrical terms are available in the three Vedas, Rg, Yajur and Atharva Veda.

The word vrttam (RV I, 155.6) is used for circular wheel of the chariot. Also trivrta (RV I, 34, 9, 12) is used for the chariot of As' vinau which was triangular in shape.

काऽऽसीत् प्रमा प्रतीमा किं निदानम् आज्यं किमासीत् परिधिः क आसीत्। छन्दः किमासीत् प्रउगं किमुक्थं यद्देवा देवमयजन्त विश्वे।। Rg. X. 130.3.

Word 'Pramā' (measure)', 'Paridhiḥ' (circumference)', 'prauga' (triangle) are all geometrical terms.

The word *catursraktih* in Yajurveda (38.20) which means a rectilinear figure with four sides.

'*Tribhuja*' occurs in Atharvaveda (VIII, 9.2), meaning a figure with three arms in triangular figure. The three *Śrauta Kuṇḍas*, namely *Gārhapatya*, *Āhavanīya* and *Dakṣiṇa*, seem to be older than Rg Veda (Oldenberg, Religion ds Veda, SBE Vol. xxx No. 2 p. 348.)

The Gārhapatya fire is mentioned by name in Rgveda:

गार्हपत्येन सन्त्य ऋतुना यज्ञनीरसि। देवान् देवयते यज।। I.15.12.

However, the shape and dimension of these three scond fires are not available in any of the Vedas.  $G\bar{a}rhapat\bar{i}ya$  of square,  $\bar{A}havan\bar{i}ya$  of circular and Daksina of semicircular in shape and of equal area can be thought of description of the  $G\bar{a}rhapatya$  as circle of one square vyama (=pursa) and  $\bar{A}havan\bar{i}ya$  of a square of same size appears in the Satapatha  $Br\bar{a}hamana$  (VII.1.1.37) which states:

व्याममात्री भवति। व्याममात्रो वै पुरुषः पुरुषः प्रजापतिः प्रजापतिरग्निरात्मसंभितां तद्योनिं करोति परिमण्डला भवति परिमण्डला हि योनिरथोऽअयं वै लोको गार्हपत्यः परिमण्डल उ वाऽअयं लोकः।

In Rgveda (I 110.5) the farm land used to be divided into equal pieces. The man during this job was called *Rbhus*.

The surveying of land was a usual matter with Aryans of Rgvedic period. Therefore it is evident they have the knowledge of geometry.

In Satpatha Brāhmaņa (I, 2.5, 1-2)

देवाञ्च वा असुराञ्च। उभये **प्राजा**यत्याः पस्पृधिरे ततो देवा ज्युव्यमिवाऽऽसुरथ हासुरा मेनिरेऽस्माकमेवेदं खलु **भुक्नामिति।।** I.2.5.1 ते होचुः हन्तेमां पृथिवी विभजामहै तां विभज्योपजीवामेति तामौक्ष्णेश्चर्मभिः पञ्चात्प्राञ्चो विभजमाना अमीयुः। I.2.5.2.

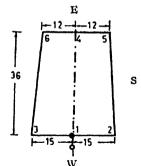
It means that among the Asuras, the unit of measurement for distribution of land was skin of bullocks. It clearly shows that to measure and distribute the land, knowledge of geometry is imperative. Satpatha Brāhmaṇa (III 5.1.26) gives the method of locating cātvāla which shows the progress in mensuration at the period.

अथ इम्यां च स्पयं चादत्ते। तद्य एष पूर्वार्ध्यः उत्तरार्ध्यः इङ्कुर्भवित तस्मात् प्रत्यङ् प्रक्रामित त्रीन् विक्रमांस्तचात्वालं परिलिखित सा चात्वालस्य मात्रा...।

It further confirms the state of knowledge from the detailed description given for the layout of the *Vedi* in *Śataptha Brāhmana* (III.5.1.1-6)

तद्य एष पूर्वार्ध्यो वर्षिष्ठः स्थूणाराजो भवति। तस्मात् प्राङ्ग प्रक्रामित त्रीन् विकमांस्तच्छङ्कुं निहिन्त सोऽन्तःपातः। III.5.1.1. तस्मान्मध्यमाच्छङ्कोः। दक्षिणा पञ्चदश विक्रमान् प्रक्रामित तच्छङ्कुं निहिन्त सा दक्षिणा श्रोणिः। III.5.1.2. तस्मान्मध्यमाच्छङ्कोः। उदङ् पञ्चदश विक्रमान् प्रक्रामित तच्छङ्कुं निहिन्त सोत्तरा श्रोणिः। III.5.1.3. तस्मान्मध्यमाच्छङ्कोः। प्राङ् षट्त्रिंञातं विक्रमान् प्रक्रामित तच्छङ्कुं निहिन्त स पूर्वार्धः। III.5.1.4. तस्मान्मध्यमाच्छङ्कोः। दक्षिणा द्वादश विक्रमान प्रक्रामित तच्छङ्कुं निहिन्त दक्षिणोंऽसः। III.5.1.5. तस्मान्मध्यमाच्छङ्कोः। उदङ् द्वादश विक्रमान प्रक्रामित तच्छङ्कुं निहिन्त उत्तरोंऽस एषा मात्रा वेदेः। III.5.1.6.

Also at the time of Śatapatha Brāhamana the Pythagoras theorem was known as the parallelism of the sites 3,2 and 6,4 could not be achived without this.



- (1) Madhyama sanku central peg
- (2) Dakśinā śroņi South-West corner
- (3) Uttarā śroni North-West corner
- (4) Pūrvārdhaḥ Estern central peg
- (5) Daksina amsa South-East corner
- (6) Uttarā amsa North-East corner

There are innumerable examples that confirm that even in pre-sulba period geometry was well known. Few of them are cited below:

- 1. The no. of bricks to construct different types of *Dhiṣṇyas* are mentioned in *Śatpatha Brāhamaṇs* for IX, 4.36-8.
- 2. The concept of construction a scrificial fire  $7\frac{1}{2}$  square pursa area is described in Śatapatha Brāhmans (X,2.3.5). It appears that geometrical knowledge was developed as per requirements of scarifical ritual, then it was converted in to Sūtraform.

The word  $Ak \not = nay \bar{a}$  which means hypotenuse or right angle triangle or diagonal of a square, rectangle or trapezium is known from the time of  $Taitriya\ Samhita$ . Also at the same time arithmetical calculations and geometrical constructions to find  $\sqrt{2}$ ,  $\sqrt{3}$  ......must have

#### been developed.

There are no. of circumstantial evidences to show that the knowledge of geometry originated from the Vedas.

Details of different types of chariots and for different purposes is clearly mentioned in RgVeda (see Rg. V56,6, II,39,4). By the time of Yajur Veda, due to increased no. of different types of chariots, separate guild of chariot makers was developed. They were treated different from carpenter.

The proficiency in production of wheels confirms a good knowledge of geometry. The fixing of Spokes of odd or even nos. require knowledge of dividing the area of the circle into desired no. of small parts of equal areas (See Rg. I.164.13).

The  $n\bar{a}bhi$  of the wheel is located at the centre of the wheel showing thereby that construction for obtaining the centre of a given circle was known at that time.

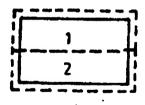
Rg Vedic Aryanas were also knowing the construction of drawing concentric circles of desired diameters, as  $n\bar{a}bhi$ , felly and the central hole must be concentric. Wells were covered by the owners (Rg. I, 55.8). Depending upon the shape of the well, knowledge of geometry is required to prepare the centre of month of the well.

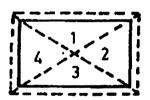
Construction of houses of different types are mentioned in Vedas, Nighaṇṭu (III.4) giving 22 names for a house in Vedas.

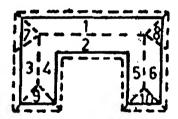
In Atharva Veda, it is clearly mentioned that a house is constructed according to measurements. A house is called *Mānasya Patnī* (Av.IX. 3.5;) also *yena chāsi mitā tvam* (Av.IX.3.9) *yas tvā sāle nimimāya* (Av.IX.3.11) etc.

The builders of the house is called *Mana-Pati*. The houses used to have many faces up to 10.

या द्विपक्षा चतुष्क्षा षट्पक्षा या निमीयते। अष्टापक्षां दशपक्षां शालां मानस्य पत्नीमग्निर्गर्भ इवा शये।। (Av.IX.3.21)







Description of sacrifice, ladies room, Kitchen etc. is clearly mentioned in Atharveda & Rgveda which confirms that geometry must have been developed at the Vedic time.

Town planning is also mentioned in Rgveda.

River-irrigation by constructing dams across them was known. The river is praised that it should flourish the country by filling the canals.

प्र पिन्वघ्वमिषयन्ती: सुराघा आ वक्षणा: पुणघ्वं यात शीभम्। (Rv.III.33.12)

Geometry seems to be necessary for the construction of even a very small dam across river.

In Atharva Veda, the town of gods called Ayodhya is described (AVX.2.31). This city should be circular in plan with eight rampart walls and nine doors.

From the above discussion, it is evident that for production of chariots, constructions of houses, and their layouts in a town and also for accurate construction of different constituent parts for a variety of sacrifices, the Aryans of the *Vedic* and *Brāhmaṇa* period might have developed the geometrical knowledge.

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# BHĀSKARĀCĀRYA: THE GREATEST SCHOLAR OF VEDIC PRIME KNOWLEDGE

#### S. L. SINGH AND RAMESH CHAND\*

In Ancient India, there had been 18 authorities (immortal beings or sages) of Vedic supreme knowledge:

सूर्यः पितामहो व्यासो वसिष्ठोऽतिः पराशरः। कश्यपो नारदो गर्गो मरीचिर्मनुरङ्गिराः।। लोमशः पुलिशश्चैव च्यवनो यवनो भृगुः। शौनकोऽष्टादशैवैते ज्योतिःशास्त्रप्रवर्तकाः।।

- 1. Sūrya, 2. Pitāmaha 3. Vyāsa, 4. Vasistha 5. Atri, 6. Parās'ara, 7. Kas'yapa
- 8. Nārada, 9. Garga, 10. Marici, 11. Manu, 12. Angirā, 13. Lomas'a or Romas'a,
- 14. Pulis'a or Paulasiya, 15. Cyavana, 16. Yavana, 17. Bhṛgu, 18. S'aunaka.

These various names stand perhaps for different schools of astronomy.

In ancient India, the word astronomy was used for the computation of celestial luminaries. What special place the astronomical culture had at that time could be visualised from the following verse of *Vedanga Jyotişa*:

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यथा शिखा मयूराणां नागानां मणयो यथा।
तद्वद्वेदांगशास्त्राणां ज्योतिषं मुर्धनि स्थितम्।।
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(As the crests of peacocks and the jewels of serpents, astronomy stays at the head of all the  $Veda\dot{n}ga$  Sastras).

Astronomy developed in ancient India as an adjunct to the rules of worship laid down in the Vedic art of living. So it came to be classified as one of the six  $Ved\bar{a}ngas$  or limbs of the Vedas and was known by the name of Jyotiṣa, which literally means the science of celestial bodies. In a secondary and later sense it stood for the study of planets, stars and heavenly bodies.

Thus astronomy stands for light and, in the highest sense, for the supreme effulgence.

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 $Bh\overline{a}skar\overline{a}c\overline{a}rya$  composed a great astronomical treatise  $\acute{S}iddh\overline{a}nta$  Siromani (SS) in 1150 A.D. at the age of 36. The SS gives a complete account of Indian astronomy up to 1200 A.D. This holy  $Siddh\overline{a}nta$  reflects the Vedic supreme knowledge of  $\overline{A}c\overline{a}rya$   $Bh\overline{a}skara$ . In this Grahaganita he says:

वेदास्तावद् यज्ञकर्म्मप्रवृत्ता यज्ञाः प्रोक्तास्ते तु कालाश्रयेण। शास्त्रादस्मात् कालबोघो यतः स्याद् वेदाङ्गत्वं ज्यौतिषस्योक्तमस्मात्।। शब्दशास्त्रं मुखं ज्यौतिषं चक्षुषी श्रोत्रमुक्तं निरुक्तं च कल्पः करौ। या तु शिक्षाऽस्य वेदस्य सा नासिका पादपद्मद्वयं छन्द आदौर्बुधैः।। वेदचक्षुः किलेदं स्मृतं ज्यौतिषं मुख्यता चाङ्गमध्येऽस्य तनोच्यते। संयुतोऽपीतरैः कर्णनासादिभिश्चनुपाऽङ्गेन हीनो न किञ्चित्करः।।

#### This means:

 $Yaj\tilde{n}a$  is fundamental to vedic culture, and the time of a particular  $Yaj\tilde{n}a$  is of vital importance. So the astronomy occupies the top place amongst the limbs of Vedas. If one compares the Vedas with a human body then Sabda Sastra (the science of grammar) is the face, astronomy stands for eyes, the *Nirukta* for ears, the *Kalpa* for hands, the Sikṣa for the nose and the *Chanda* Sastra for legs. Since a body without eyes is hollow, the astronomy remains the top most limb of the *Vedapuruṣa*.

### LIFE HISTORY OF BHASKARACARYA

The great astronomer, astrologer and top-notch mathematician saw the light in Saka 1036 (1114 A.D.) at the village Bid (Presently known Bijapur in Karnataka). The village is located near mountain Sahya, famous for the central place of divine scholars in those days. He was born in Sandilya Gotra (lineage). His father Maheśvara was a great scholar and priest. His wisdom was hereditary as he belonged to a wise race and scholarly family. He himself has written in Golādhyāya of SS:

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रसगुण पूर्णमही समशकनृपसमयेऽभवन्ममोत्पत्तिः।
रसगुणवर्षेण मया सिद्धान्तशिरोमणी रचितः।।
आसीत् सह्यकुलाश्रितपुरे त्रैविद्यविद्वज्जने।
नानासज्जनधाम्नि विज्जडविडे शाण्डिल्यगोत्रो द्विजः।।
श्रौतस्मार्तविचारसारचतुरो निःशेषविद्यानिधिः।
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साधूनामवधिर्महेश्वरकृती दैवज्ञचूड़ामणि:।।
तज्ज्स्तच्चरणारविन्दयुगलप्राप्तप्रसादः सुधी:।
मुग्धोद्वोधकरं विदग्धगणकप्रीतिप्रदं प्रस्फुटम्।।
एतद्व्यक्तसदुक्तियुक्तिबहुलं हेलावगम्यं विदाम्।
सिद्धान्तग्रथनं कृबुद्धिमथनं चक्रे कविभास्करः।।

The following inscription (of about 1210 A.D.) due to Cangadeva, the grandson of  $Bh\overline{a}skar\overline{a}c\overline{a}rya$  was first discovered by Dr. Bhau Daji at the village Patana near Devgiri (Sivanatha Jharakhandi,  $Bh\overline{a}rt\overline{i}ya$  Jyotişa):

शाण्डिल्यवंशे कविचक्रवर्ती त्रिविक्रमोऽभत्तनयोऽस्य जात:। यो भोजराजेन कृताभिधानो विद्यापतिर्भास्करभट्टनामा।। गोविन्दसर्वज्ञो गोविन्दसन्निभ:। तस्मात जातो प्रभाकर: सृतस्तस्मात् प्रभाकर इवापर:।। तस्मान्मनोरथे पर्णमनोरथ:। जात: सतां श्रीमन्महेश्वराचार्यस्ततोऽजनि कवीश्वर:।। तत्स्न: कविवन्दवन्दितपद: सद्वेदविद्यालता कन्दः कंसरिपुप्रसादितपदः सर्वज्ञविद्यासंदः। यच्छिष्यै: सहकोऽपि नोविवदितुं दक्षो विवादी क्वचित् श्रीमानुभास्करकोविद: समभवत् सत्कीर्तिपुण्यान्वित:।। वेटार्थवित्तार्किकचऋवर्ती। लक्ष्मीधराख्योऽखिलसुरिम्ख्यो **क्रतुक्रियाकाण्डविचारसारविशारदो** भास्करनन्दनोऽभूत।। सर्वशास्त्रार्थटक्षोऽयमित्रि प्रादत्त:। मत्वा: **जै**त्रपालेन विवधाग्रणी।। भीत: कृतश्च सत: सिंघणाचऋवर्विदैवयज्ञवर्योऽजनि चंगदेव:। श्री भास्कराचार्यनिबद्धशास्त्रविस्तारहेतोः कुरुते मठं यः।। ग्रन्था: सिद्धान्तशिरोमणि प्रमुखा:। रचित भास्कर कृताश्चान्ये व्याख्येया मन्मठे नियमात्।। तद्रंश्य

The following is the genealogy of Bhaskaracarya:

 $Trivikarama \rightarrow Bh\overline{a}skarabhata \rightarrow Govinda \rightarrow Prabh\overline{a}kara \rightarrow Manoratha \rightarrow Maheśvara \rightarrow Bh\overline{a}skar\overline{a}c\overline{a}rya \rightarrow Lakṣhmidhara \rightarrow Caṅgadeva$ .

#### **EDUCATION**

Bhāskarācārya received education at the feet of his learned father since his childhood. He studied Vedas, Purāṇas, Upaniṣadas, Rāmāyaṇa, Mahābhārata, Gītā, Siddhāntas and several other works of Varāhamihira, Brahmagupta, Śridhara, Lalla, Padmanabha, Pṛthudakasvami, Munjāla and Śrīpati and others. He offered the humble criticism in the works of his forescholars, whenever he found errors. For, example, he criticised Lalla in the following stanza of Golādhvāya:

दुष्टं कन्दुकपृष्ठजालवदिलागोले फलं जल्पितं लल्लेनास्य शतांशकोऽपि न भवेद्यस्मात्फलं वास्तवम्। तत्प्रत्यक्षविरुद्वमुद्धतिमदं नैवास्तु वा वस्तु वा हे प्रौढा गणका विचारयत तन्मध्यस्थबृद्ध्या भृशम्॥

(Pt. Kedardatta Joshi (Ed.) Golādhyāya of Bhāskarācārya, Motilal Banarsidas, Delhi, 1988).

The Following stanza shows the deptth of his knowledge:

अध्ये व्याकरणानि षट् च भिषजां व्याचष्ट ताः संहिताः षट् तर्कान् गणितानि पञ्च चतुरो वेदानधीते स्म यः। रत्नानां त्रितयं द्वयं च बुबुधे मीमांसयो रन्तरं सदुब्रहमै कमगाधबोधमहिमा सोऽस्याः कविर्भास्करः।।

This means: It was the poet  $Bh\overline{a}skara$ , who had read all the eight volumes of grammar and the six branches of medical science with their commentaries, who had studied the six logics, five branches of mathematics and the four Vedas, a trial of three ratnas, who could distinguish between the two  $Mim\overline{a}ns\overline{a}s$  (branches of philosophy), only the great  $Brahm\overline{a}$  (creater of the universe) could know the depth of his knowledage. [Thus he had not mustered only Mathematics but was well versed in many branches of knowledge.] Also:

भास्करीय गिरां सारं भास्करो वा सरस्वती। चतुर्मुखोऽधवावेति विदुर्नान्ये तु मादुशाः।।

**This means:** The essence of the words of  $Bh\bar{a}skara$  could be understood either by  $Bh\bar{a}skara$  himself or by  $Sarasvat\bar{i}$ , the goddess of learning or by  $Brahm\bar{a}$ , the creater of the universe, and not by (ordinary persons like us).

#### STYLE

In the beginning of his Grahaganita he says:

कृती जयित जिष्णाजो गणकचऋचूड़ामिण— र्जयन्ति लिलतोक्तयः प्रिथततन्त्रसद्युक्तयः। वराहमिहिरादयः समवलोक्य येषां कृतीः कृती भवित मादृशोऽप्यतन्तुन्त्रबन्धेऽल्पधीः।। This shows that  $Bh\overline{a}skar\overline{a}c\overline{a}rya$  was influenced with the poetic style of Brahmagupta and  $\overline{A}c\overline{a}rya$  Var $\overline{a}$ hamihira. He followed their poetic style in his writings, and the climax of his poetic beauty is attained in his  $L\overline{i}l\overline{a}vat\overline{i}$  WORKS  $Bh\overline{a}skar\overline{a}c\overline{a}rya$  was inspired by the holy sermon of  $Git\overline{a}$ :

कर्मण्येवाधिकारस्ते मा फलेषु कदाचन्।

(Vināyaka Gaņesa Āpate, Bījagaņita of Bhāskarācārya Ānanda Āsarama Press, Poona, 1930). He composed Siddhānta Śiromaņi without any imperial support. His commentators also follwed him. His SS is a comprehensive work in four parts.

- 1. LĪLĀVATĪ (arithmetic, algebra, gemoetry, mensuration, and a bit of trigonometry): It contains about 280 verses including half verses.
  - 2. BIJAGANITA (algebra): It contains 187 verses.
- 3. GOLĀDHYĀYA (theory of sphere or trigonometry inculding spherical trigonometry.) It deals with the astronomical basis of mathematical calculations about grahas (planets). It contains 563 verses.
  - 4. GRAHAGANITA (planetary motion): It contains 447 verses.

#### HIS OTHER WORKS

- 1. KARAŅA KUTŪHALA: It is an astronomical work containing 138 verses. He composed this book in śaka 1105 (1183 A.D.) when he completed the age of 69 years.
  - 2. SARVATOBHADRA YANTRA
  - 3. VASITHTULYA

#### BHASKARACARYA: THE GREAT ASTRONOMER

Spottiswoode (S.R. Sinha, Mathematics Education (Siwan), 15 (1981), p.82) remarks:

It must be admitted that the penetration shown by *Bhaskara* in his analysis is in the highest degree remarkable, that the formulae which he establishes and the methods of establishing them bear more than a resemblance ..... they bear an analogy ..... to the corresponding process in modern astronomy, and the majority of scientific persons will learn with surprise the existence of such a method in the writings of so distant a period and so distant a region.

Golādhyāya and Grhaganita of  $Bh\bar{a}\,skar\bar{a}\,c\bar{a}\,rya$  give a complete account of Indian astronomy nearly up to 1200 A.D. A thorough discussion of the obliquity of the ecliptic and its effect on the equation of time reflects a great astronomical maturity of the  $\bar{A}\,c\bar{a}\,rya$ .

For the measurement of time  $Bh\bar{a}skara$  gives an account of time scale yuga system as described and used by ancient Indian seers and astronomers. He gives the following stanza for the computation of planetary positions of the planets, when the present Kaliyuga begins.

खाद्रिरामाग्नयः ३३७० क्कग्निरामाङ्कका ९३३१ वेदवेदाङ्कचन्द्रा १९४४ विलिप्ताः क्रमात्। षड्साङ्गब्धयो ४६६६ ऽङ्गभ्रवेदाब्धयो ४४०६ वेदषट्काभ्रभूपाभ्रभूसंमिताः १०१६०६४।। वेदचन्द्रद्विवेदाव्धिनागाः ८४४२१४ कर— द्व्यव्धिवेदाव्धिशैला ७४४४२२ भवेयुः कुजात्। द्वापरान्तधु वाश्चक्रशुद्धास्तथा सूर्यतुङ्गेन्दुतुङ्गेन्दुपातोद्धवाः।।

#### This means:

Mars	Mercury	Jupiter	Venus	Saturn	Solar Apogee	Lunar Apogee		ending r node
11R	11R	11R	11R	11R	2 R	4 R	5R	Rāsis
29°	27°	29°	28°	28°	17°	5°	3°	Degrees
3'	24'	27'	42'	46'	45'	29'	12'	Minutes
50"	29"	36"	14"	34"	36"	46"	58"	Seconds

For the computation of ahargana cahargana literally means aggregate of civil days) since the beginning of creation up to the end of Saka king he gives the following verse in his Grahaganita:

याता षण्मनवो युगानि भिमतान्यन्यद्युगाङ्घ्रित्रयं नन्दाद्रीन्दुगुणा (३१७९) स्तथा नृपशकस्यान्ते कलेवंत्सराः। गोद्रीन्द्वद्रिकृताङ्कदस्रनगगोचन्द्राः (१९७२९४७१७९) शकाब्दान्विताः सर्वे सङ्गलिताः पितामहदिने स्यूर्वर्तमाने गताः।

Secondly, he gives the following verse for the computation of ahargana.

कथितकल्पगतोऽर्कसमागणो

रविगुणो गतमाससमन्वितः। खदहनै ३० गुणितस्तिथिसंयुतः पृथगतोऽधिकमास १५९३३०००० समाहतात्।। रिविदेना १५५५२००००००० सगताऽधिकमासकै: कृतिदेनै: सिहतो द्युगुणो विधो:। पृथगतः पिठतावम २५०८२५५०००० संगुणाद् विधृदिना १६०२९९९००००० सगतावमवर्ञ्जितः।। भवित भास्करवासरपूर्व्वको दिनगणो रिवमध्यमसावनः। अधिकमासदिनक्षपशेपतो द्युष्टिकादिकभत्र न गृह्यते।।

In his  $siddh\bar{a}nta$  Siromani,  $Bh\bar{a}skar\bar{a}c\bar{a}rya$  has given several techniques for the computation of planetary positions at any moment. He gives a wonderful verse for the computation of planetary position of all the planets:

दिग्भिः १०१४६१ नगाष्ट्रनगभूतिथिभिः क्रमेणा १५१,७८७। देवाष्ट्रखाङ्कशशिभिः १९०८३३ च रसाग्निवेद— सिद्धैः २४४३६ खखाब्धिदहनाभ्रयमेन्दुभिश्च १२०३४००।। भूपाब्धिलोचनरसैः ६२४१६ खखखाभ्रनन्द— नन्दाश्विभिः २९९०००० गगनखाभ्रगजाङ्कनागैः ८९८०००। खाभ्राष्ट पङ्गजधृतिप्रमितैः १८८६८०० च भक्ताद् भागादिकानि हि फलानि रवेः राकाशात्।। विघोः फलं खाश्विगुणं विधेयं ग्रहधुवाः स्वस्वफलैः समेताः। ते वा भवन्ति दुचराः क्रमेण भागादिकः स्यात् फलमेव भानुः॥

For the computation of solar and lunar eclipses he has given scientific methods.

#### BHASKARACARYA: THE GREATEST MATHEMATICIAN

 $Bh\overline{a}\,skar\overline{a}\,c\overline{a}\,rya$  was one of the greatest mathematician who appears to have established first the modern theory in the convention of signs (minus x minus = plus) in the following verse:

स्वयोरस्वयो: स्वं वध: स्वर्णघाते क्षयो भागहारेऽपि चैवं निरुभत्तम्। (A study of of *Bhāskarācārya*, Chaukhamba Orientaliya, Varanasi, 1979, p.51).

Bhā skarā cā rya, in Līlāvatī, gave formulae for the surface area and volume of sphere and volume of frustum of a pyramid. His logistics include addition, subtraction, multiplication, division, squaring, cubing, square-root, cube-root of numbers. It also deals with the algebra of zero, the rule of three (which was later called Golden rule in the West) permutations, combinations and Kuṭṭaka. The section on geometry deals with triangles, quadrilaterals, circle, cyclic sphere, trigonometricals functions etc.

In  $Lil\bar{a}vat\bar{i}$   $Bh\bar{a}$   $skar\bar{a}$   $c\bar{a}$  rya picked up the nature as the store house of wisdom. The following stanza reflects  $Bh\bar{a}$  skara s love to nature while posing a question on quadratic equation:

यातं हंसकुलस्य मूलदशकं मेघागमे मानसं प्रोइडीय स्थलपद्मिनीवनमगादष्टांशकोऽम्भस्तटात् बाले बालमृणालशिलिन जले केलीक्रियालालसं दृष्टं हंसय्गत्रयं च सकलां यथस्य सङ्ख्यां वद।।

#### That is:

O! tender girl, on the advent of rains out of the swans in a lake, ten times the square-root of their number went to *Manasarovar*, 1/8th (of them) moved to Sthala Padmini (a forest). The remaining three pairs were engaged in water sports. Tell the total number of swans.

We fully share what Datta and Singh say: It is certain, however, that after the 12th century very little origional work was done in India, commentaries on older works were written and some new works brought out, but none of these had sufficient merit as regards exposition or subject matter, so as to refine and extend the works of *Bhāskara* II, which had held undisputed sway for almost nine centuries (as standard text books).

 $Bh\overline{a}\,skar\overline{a}\,c\overline{a}\,rya$ 's  $B\overline{i}jaganita$  is akin to our present day secondary school algebra and a little more. The  $B\overline{i}jaganita$  gives general methods for solving linear quadratic indeterminate equations which were given after 500 years by western scholars-such as Fermat (1601-1665), Lagrange (1736-1813) and Euler (1707-1783).  $Bh\overline{a}\,skara$  gave the  $cakrav\overline{a}\,la$  method for obtaining the general integral solution of the indeterminate equation  $Nx^2 + 12 = y^2$ . This is considered a big achievement in Number Theory during the golden period.

 $Bh\bar{a}\,skar\bar{a}\,c\bar{a}\,rya\,$  gave the concept of mathematical infinity (Khahara) and the analysis concerning the limit of an algebraic expression is implicited in his examples in  $L\bar{i}l\bar{a}vat\bar{i}$  and  $B\bar{i}jaganita$ . In the history of mathematical sciences, this is perhaps the greatest contributions to the human civilization.

Martin Ohm (1628) says: 'If a is not zero but b is zero, then the quotient  $\frac{a}{b}$  has no meaning for 'the quotient multiplied by zero gives only zero and not a, as long as a is not zero" (Datta and Singh, History of Hindu Mathematics, p. 246). That is even 19th century modern mathematicians were not clear in such concepts.

Indeed, it appears that Bhaskaracarya was the first to introduce the concept

of limit. He has used this technique in his astronomy.

The recent investigations by S.R. Sinha. [The Mathematics Education (Siwan) 15 (1981), 69-82] and C.N. Srinivasienger [History of Ancient Indian Mathematics, World Press Private Ltd., Calcutta 1967] have shown that  $Bh\bar{a}skar\bar{a}c\bar{a}rya$  developed the concept of Differential Calculus but certainly not in the modern language. The above notion, 500 years before Newton and Leibniz appears in his famous treatise SS. Indeed, for the exact computation of daily motion of planets,  $Bh\bar{a}skara$  introduced the concept of  $Tatk\bar{a}likagati$  (instantaneous velocity) by dividing the day into a large number of small intervals and compared the positions of the planets at the end of successive intervals.

If y and y' are the mean anomalies of the planets at the end of concecutive intervals, then according to  $Bh\bar{a}skara$ :

बिबार्धस्य कोटि ज्यागृण स्म्निज्याहरः फलं दोर्ज्यायोरंतरं।

(The product of the cosine of the semidiameter by the element of the radius gives the difference of two sines). This result is equivalent to:

$$H \sin y' - H \sin y = (y' - y) H \cos y$$

In modern language, this is equivalent to

$$\delta(\sin y) = \cos y \, \delta y$$

In relation to the basic idea of ROLLE'S THEOREM  $Bh\bar{a}\,skar\bar{a}\,c\bar{a}\,rya$  has gone deeper into Differential Calculus in SS and suggests that the differential coefficient vanishes at an extreme value of the function. In Grahaganita ( $spast\bar{a}dhik\bar{a}ra$ ) he says:

(where the planets motion is an extremum, there the fruit of motion is absent, i.e., the motion is stationary. At the commencement and end of retrograde motion, the apparent motion of the planets vanishes). That is, the derivative vanishes at a point of maxima or minima.

#### BHA SKARA CARYA: THE GREATEST SCIENTIST

Regarding the gravitational power of the earth  $Bh\overline{a}skar\overline{a}c\overline{a}rya$  criticised the Buddhist philosophy (Pt. Kedardatta Joshi (Ed). Goladhyaya of  $Bh\overline{a}skar\overline{a}c\overline{a}rya$ . Motilal Banarsidas, Delhi, 1988) and gave the appropriate theory in the following stanza of his  $Gol\overline{a}dhy\overline{a}ya$ :

"आकृष्टिशक्तिष्ठ महीतया यत् खस्थं गुरुंस्वाभिमुखं स्वशक्तया। आकृष्यते तत्पततीव भाति समे समन्तात् क्र पतित्वयं खे॥ "

("पृथ्वी में गुरूत्वाकर्षण शक्ति है जो आकाश के गुरू पदार्थों को खींचकर अपने धरातल में ले आती है। पृथ्वी में यह एक ईश्वरदत्त गुरूत्वाकर्षण शक्ति सदा वर्तमान रहती है, जो आकाश में अपनी सीमा के भीतर भी किसी भी गुरू पदार्थ को खींचकर अपने धरातल में ले आती है और साधरण द्रष्टा को आकाशीय पदार्थों का पृथ्वी में गिरने का सा अनुभव होता है"।)

#### THE POET

The SS reflects that  $\overline{A}$  carya was keenly interested in poetry. He describes Rtuvarnanam and Srigonnativasanadhyaya in Goladhyaya in the form of poetry which is embalished with figures of speech such as simile, metaphor, and hyperbole etc. In  $L\overline{i}l\overline{a}v\overline{a}t\overline{i}$ , he has given several examples from the nature, which show that he understood nature as the store house of wisdom. Indeed he was a great lover of nature.

#### **DEATH**

 $Bh\overline{a}\,skar\overline{a}\,c\overline{a}\,rya$  (Trans.), the great mathematician, astronomer and astrologer left for his heavelny abode in 1193 A.D. (K.S. Patavardhan, S.A. Naimpally and S. L. Singh,  $L\overline{i}l\overline{a}vat\overline{i}$  of  $Bh\overline{a}\,skar\overline{a}\,c\overline{a}\,rya$ , Motilal Banarsidas, Delhi 2001). Even we Indian foregot to celebrate the 800th anniversary of the 'SIDDHĀNTA ŚIROMANI' which is one of the greatest astronomical works on this earth.

We assest without any hesitation that Bhāskarācārya was a great mathematician, astronomer, scientist, philosopher, poet and a great scholar of Vedic prime knowledge and he made significant contributions to lay the foundation stone of modern mathematics, modern astronomy, arithmetic, algebra, trigonometry, geometry, calculus and the concept of limit. He will remain the source of knowledge and inspirations for mathematicians and the generations to follow.

#### **Book Review**

by V. Madhukar Mallayya (Trivandrum)

#### LĪLĀVATĪ OF BHĀSKĀRĀCARVA

A Treatise of Mathematics of Vedic Tradition

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Bhāskara II or Bhāskārācārya was one of the most illustrious astronomer-cum-mathematician of the 12th Century A.D. The Ācārya has Composed standard treatises on astronomy, mathematics and astrology, namely the Siddhānta Śiromani-Lilāvatī-Bījaganita, Vāsanābhāsya, Karanakutūhala, Bhāskara Vyavahāra and Vivahapatala. Of these, the Siddhānta Śiromani-Lilāvatī-Bījaganita combine from the magnum opus of the Acārya. Because of the high merit of the contents, the works of the Acārya became the main source of reference for mathematicians and astronomers of later period. Most of the other works composed during that period were over shadowed and driven to obscurity by the remarkable brilliance of Bhāskāra's analytic exposition.

The Lilavati is a basic treatise on Mathematics encompassing the pre-requisites for a wider study of astronomical science and various topics useful for domestic computations having applications in day to day life. This delightful treatise was composed by Bhāskara at the age of 36 in the year 1150 A.D. Lilāvati in Sanskrit means 'charming' or 'beautiful'. As such the title of this charming work of mathematical enunciations and beautiful illustrations is quite appropriate. Faizi's translation of the work into Persian narrates a touching story relating to a misfortune that be fell on the beautiful daughter Lilāvati of Bhāskarācārya on account of which she had to remain unmarried for ever in her life. In order to console his unfortunate

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daughter and keep her dejected mind fully occupied in learning mathematics, the intelligent father started teaching her mathematics from the basic level in an interesting manner and composed the quintessence of his deliberations in the form of beautiful verses and named it Lilāvatī. On completion of this task, the Ācārya seems to have totally succeeded in bringing back joy and happiness to his daughter and probably because of this experience Bhāskara generalizes and happily remarks in the concluding stanza that anyone who masters the Lilāvatī will become happy and prosperous. This message is embodied in the verse-

येषां सुर्जातगुणवर्गभूषितांगी शुद्धाखिलव्यवहृतिः खलु कंठसक्ता। लीलावतीह सरसोकितमुदाहरन्ती तेषां सदैव सुखसंपद्पैति वृद्धिम्।।

"Joy and happiness is indeed ever increasing in this world who have Lilavati clasped to their throats, decorated as numbers are with neat reduction of fractions, multiplications and involutions, pure and perfect as they are the solutions and tasteful as in speech which is exemplified.

The book under review is an enlargement of the Phadke's Marathi version of the Lilavati, translated into English and expounded in modern mathematical language. The work is divided into thirty four chapters followed by the index of verses and a subject index.

Bhāskara's first invocation stanza to lord Ganesa is prefixed to Chapter 1. After invoking the blessings of the Lord, Bhāskara proceeds to describe the units of value (currency denominations) that was in vogue in the country during that period, followed by tables of commonly used weighing and measuring units. An understanding of size and weights of objects and measures of space around is a basic need of man and absolutely essential in the science of computation. Knowing fully the all pervasive nature of measures, the section on weights and other measures has been separately dealt with and prefixed to the actual treatise by the Ācārya. The authors of the book under review have given in Chapter 1, their detailed comments on each of the measures listed down by Bhāskara. The actual mathematical treatise commences with Bhāskara's second invocation stanza, followed by the numeration terminology for dealing with numbers running upto 18 places of figures. The

contents of the book from Chapter 2 to Chapter 34 may be broadly classified into three sections.

- i) Ankaganita dealing with real numbers and various operations and determinations involving numbers.
- ii) Ksetraganita dealing with the geometrical figures both plain as well as solid along with some of the applications in domestic computations, and
- iii) Kuttaka dealing with indeterminate equations of first degree and methods of finding their solutions. Without any such classification, the various topics coming under the preview of these three are divided into thirty three chapters and discussed briefly in the modern language by the authors. These Chapters are entitled, Place Value of Digits; Addition and subtraction; Method of multiplication; Division; method of finding squares; Square root; Method to find the cube; Cube roots; Eight operations on fractions; Addition and subtraction of fractions; Multiplication of fractions; Division of fractions; Squares, Cubes, Square roots and Cube roots of fractions; Eight rules concerning zero; Reverse process; To find an unknown quantity; Method of transition; Square transition; Quadratic equations; The rule of three; Inverse proportion; The rule of five; Rule for barter; Simple interest; Combinations; Progression (Series); Mensuration; Volume; Wood cutting; Volume of a heap of grain; Shadows; Pulverization; Concatenation (Permutation, Partitions etc.). The authors have added their own comments after giving translation of various verses in simple form in lucid English, the style of which is easy to follow and any student interested in this field can get a clear idea in the modern terms of the content of the verses. However, in the case of some of the verses the authors have deliberately done away with literal translations and the renderings of such stanzae give only mathematical formulae by expressing the quintessence of the content of Bhāskara's enunciations in an intelligible form using modern mathematical symbolic notations. The authors have succeeded in their efforts to bring out clearly and accurately the mathematical knowledge embedded in the words of Bhaskara and they have expounded the contents in modern language in an appreciable form which will definitely enable the modern reader to study the sutras without any strain and without loss of Bhaskara's efforts to make the learning of mathematics interesting and charming. The translators efforts to make "faithfulness" and "beautifulness" to go together, by narrowing down the gap between Sanskrit verses and their English

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renderings without drifting far away from the literal beauty of the verses and at the same time remaining deeply faithful to the mathematical message latent in them, is quite appreciable. For instance, Bhāskara's analysis of indeterminate equations is fully explained in modern language using continued fracions. To mention another noteworthy message, the Acārya's rules concerning division by zero is explained using modern concept of limits thereby making one hear the knocking at the doors of calculus and infinitesimals by the Ācarya as early as 12th century AD. However historical remarks and notes are not many and if such comments are also included in the next edition of the book, it will be widely appreciable. The natural order of words in a verse (anvaya) may also be given prior to their English translation so that the readers having some knowledge of Sanskrit can enhance their enjoyment of learning basic mathematics by riding smoothly through the medium of poetry and thus appreciate fully the beautiful poetic flights of a scientific literature.

The Pubalishers of the book also deserve congratulations for their nice job in bringing out the book in neat printing, free from errors, with overall beauty in get up and lay out. This modern rendering of the Lilavati with brevity in words of mathematical wisdom maintaining throughout the clarity in conveying the mathematical message of Bhaskara continues to give pleasure and insight to every class of readers from children to the aged who are interested in both Sanskrit and Mathematics.

So, dear reader, do read this charming work, own a copy and adorn your personal library which will be a valuable addition to your own personal collections in the sense that, some day, some child, some where who happens to read the delightful Lilavati of Bhāskarācārya, may get inspired and motivated by the charm of the words of Bhāskarā II and hopefully a new Bhāskarācārya (Bhāskara III) may emerge in this Land of Wisdom which has bequeathed to us a bright galaxy of eminent astronomers and mathematicians.

### Abstracts of Papers/Talks

# NATIONAL SEMINAR on HISTORY OF MATHEMATICAL SCIENCES AND APPLICABLE MATHEMATICS

7-9 MARCH 2003

**Department of Mathematics and Statistics** 

GURUKULA KANGRI VISHWAVIDYALAYA HARDWAR 249 404

#### 1 YAMATAIZATION

#### B. S. Yadav

Delhi University, Delhi

Every language has a grammar of its own. Out of all languages, it is well known that Sanskrit has the most elaborate and precise grammar. Even its poetical composition is metrical. Sanskrit verse is regulated by quantity and not by accent. Though Agni Purana deals with the subject exhaustively, the Pingalachhandas-sastra by Pingalacharya has been in popular use. In this case, chhands are meterd by eight Ganas or syllabic feet. There is a *sutra* (formula): yamatarajabhansalagam which serves as a handy tool to memorize these eight ganas. The talk concentrates on the history and composition of this formula with various ramifications in mathematics, particularly with its connection with Memory Wheels and other solved and unsolved problems. Yamataization is a name coined to present the subject in a more general framework.

### 2 CALENDER SYSTEM IN ANCIENT INDIA

#### G. S. Pandey

105, Sant Nagar, Ujjain.

Ever since the dawn of Vediec civilization on this planet the counting of time has considered as the highest need of the society. During the Rigvedic period a solar year was recognized as formed of twelve months and each month of 30 days. Vedang Jyotisa (1400 B.C.) introduce the concept of Yuga system of five samvatsars (years) and made a clear distinction between a tithi (lunar) day, Savan and Solar day. Also, in order to account for a solar year, the system of intercalary month was introduced.

We present a short account of the main fifteen-calendar system introduced in India from time to time. In particular, we trace the history of Vikrma Samvat (ignited from 57 B.C.) formulated on the basis of Surya Siddhanta, which is the only calendar system being used, with some variations, almost everywhere in India, while the Government of India has declared an entirely fabricated solar form of Saka Samvat as the national Calendar.

#### 3 BAHAUDDINAL-CAMILI 'S ESSENCE OF MATHEMATICS-AS A TEXT BOOK IN MADRASAS

#### S. M. R. Ansari

Roshan Villa, Muzammil Manzil Compound, Dodhpur, Aligarh

Al-cAmili was an Iranian mathematician of  $16^{th}$  century. His Arabic treatise: Khulas\*atul H'isab was very popular in all Iranian and Indian Madrasas. It was translated into Persian in Medieval India, and also translated into German and French in the first quarter of  $20^{th}$  century. The Russian translation is also now available. Many commentaries were written in Persian by Medieval Indian scholars. This is just a brief introduction. The significance of the work is also due to the fact, that al-cAmili was the first mathematician who stated in his book the so-called Fermat theorem:  $x^n + y^n = z^n$ ? for n>2.

### 4 MAHAVIRA'S METHOD OF FALSE POSITION

#### Kanahiya Jha

Dep. of Mathematics, L.B.S.M College, Jamshedpur

History is essential for us, so that we can assess, how much we are indebted to our fore fathers. The great master has given us different ideas. If we develop these ideas to suit our needs it will be a real salutation to them. For this we have to translate these ideas in modern terms, so that it can be fit for use in solving modern problem. We will discuss briefly some ideas and rules of Mahavir (9th Century) from his work ganit sara sabgrah, about rule of false position, method of obtaining unit fraction for any given fraction some interesting aspects of series etc.

## 5 MATHEMATICS IN VEDIC TRADITION Virendra Arora

Dep. of Mathematics & Statistics, GKV, Hardwar

This talk aims at presenting a brief overview of mathematical activity in Vedic literature.

## 6 NEW DIRECTION IN VEDIC MATHEMATICS

#### S. L. Singh

Dep. of Mathematics & Statistics, GKV, Hardwar

The purpose of this talk is to identify some new components of mathematics of Vedic tradition, which has relevance with modern taste in mathematical sciences.

## 7 SIMILIRITY OF PLANE FIGURE AND GEOMETRIC AND GROUP THEORETIC STUDY OF CYCLIC QUADRILATERAL OF FIRST KIND

#### Vinod Mishra and S. L. Singh

Dep. of Maths, Sant Longawal Inst. of Engg. & Tech., Longowal, Punjab.

Dep. of Mathematics & Statistics, GKV, Hardwar

Similarity of plane figure is a very old concept. The approach is to study the valuable contributions by the Arabs, Babylons/Summerians, Chinese and Greeks with reference to the Indians at large and importance of the concept as such. Finally, we study cyclic quadrilateral of first kind geometrically. Its group theoretic study concludes with a non-abelian group of eight permutations in which four permutations are equal to another four.

### 8 SOME INDIAN SAVANTS OF MATHEMATICS

#### Man Mohan and Subhash Chander

Dep. of Mathematics & Statistics, Ramjas College, Delhi E-mail: Yudianshm@yahoo.com, subchanda@yahoo.co.in

Thousands of years ago, Indian mathematicians authored various theses and dissertations on mathematics. It is now commonly believed and widely accepted that these treatises laid down the foundations of algebra, algorithm, square roots, cube roots, various methods of calculation, and the concept of zero. It is also believed, that in Vedic times mathematical formulae were often taught within the context of spiritual expression (mantra) and thus while learning spiritual lessons, one could also learn mathematics. Here we present a brief account of 38 Indian savants of Mathematics covering the period 800 BC-1980 AD. Most of the information is based on historical resources available on the Web and arranged in the chronological order of their known or guessed periods.

## 9 A BRIEF SURVEY OF ANCIENT MATHEMATICS – A SHORT NOTE ON COMMERCIAL MATHETMATICS

#### J. Achari

Dep. of Mathematics, Statistics and Computer Science NES, Science College Nanded (Maharastra)

#### 10 GOLDEN PEROID OF INDIAN MATHEMATICS

#### B. B. Kulkarni

Netaji Subhas Chandra Bose College, Nanded (Maharastra)

## THE KRIYAKRAMAKARI EXPOSITION ON MADHAVA SERIES APPROXIMATIONS

#### Madhukar Mallayya

Chandni, TC 25/1975, Deshabhimani Road, Trivandram (Kerala)

The Kriyakramakari is a sixteenth century commentary in Sanskrit on the famous Lilavati of Bhaskaracarya. Commenting on Bhaskara's enunciation for determination of gross and nearly precise circumference (suksma paridhi and sthula paridhi) of a circle, the commentator Sankara Varrier gives an elaborate and excellent exposition on this topic by incorporating several methods for getting better and better approximations attributed to his ancestors. Tremendous achievements made by Sangamagrama Madhava of fourteenth century in the field of infinite series and series approximations are highlighted in this section of the commentary. The commentator refers to various methods based on infinite series and gives an excellent demonstration of them by employing certain ingenious techniques of modern analysis. Sankara gives an interesting derivation of Madhava's correction functions for approximating certain series. Moreover, various other infinite series are found to have been deduced from these correction functions

### 12 INNOVATIONS IN KERALA ASTRONOMY

#### S. Madhavan

40/513, 3rd Puthen St, Trivandrum

In the post Bhaskara.period, Kerala has been a centre of stormy developments in mathematics and astronomy. Some of the innovations are described here. First of all, the concept of Jya and the table of Jyas introduced by Aryabhata, Varahamihira and others underwent drastic changes. Madhava found the series for PI and changed the figure 3438' for the radius or trijya as 3437' 44" 48". He also found the infinite series for R sine and R cosine. The proof given in Yuktibhasa abounds in novel concepts.

Madhava's amendment of Chandravakyas and his interesting method of finding the moon's position using the properties of periodic functions are strikingly original. The methods of successive approximation used in various contexts also deserve investigation. Some results in Karapaddhathi and Puthumana Somayaji and Satratnamala of Sankaravarman indicate good knowledge of spherical trigonometry.

### 13 MEASURE THEORY – HISTORICAL PERSPECTIVE

#### Pramila Srivastava

Allahabad Mathematical Society, 10, C.S.P. Singh Marg, Allahabad-211 001

The purpose of this talk is to present a development and discuss its evolution.

#### 14 ALGEBRA OF NĀRĀYANA

#### S. L. Singh and M. M. Dwivedi

Dep. of Mathematics and Statistics, GKV, Hardwar Mahamati Pran Nath Mahavidyalaya, Chitrakoot

The first author has recently found Ganitavatansa (c.1356. A.D.) of Nārāyaṇa Paṇḍita from Mr. Vikas Ghosal of West Bengal. The authors have under taken the study of this rare text-book of algebra, which is hopefully akin to the algebra of Bhāskara II. The purpose of this talk is to present some glimpses from the same.

## 15 RELEVANCE OF HISTORY IN MATHEMATICS TEACHING

J. M. C. Joshi

Kumaun University, Nanital, 47 A Heera Nagar, Haldwani

#### 16 PLACE VALUE SYSTEM IN TRADITIONAL ALGEBRA

#### Ramesh Chand

Government Sarvodya Bal Vidyalaya, B-Block, Nand Nagri, Delhi

In this paper an attempt is made to discuss the concept of place value system in algebra.

#### 17 FRACTALS IN LĪLĀVATĪ

#### Mamta Rani

Dep. of Computer Sc. & Engg., SRM CEM, Faizabad Road, Lucknow

Fractal theory, a newly developed branch of computer graphics is based on feedback system. However, this field has been recognized three decades back but its germs are found in mathematics of Vedic tradition. One of the classical fractals has been found in Bhāskarācārya's Līlāvatī known as *Meru*, generally called Pascal triangle. Some recently developed fractals are also akin to figures found in ancient Indian compositions.

### 18 THE PENDULA IN MATHEMATICAL SCIENCES

#### R. S. Kaushal

Dep. of Physics & Astrophysics, University of Delhi

The role played by the pendulum and its variants in the development of mathematical sciences in reviewed. In particular, the case of a pendulum, viz., harmonic oscillator in physics is highlighted in the context of mathematical, conceptual, conventional and engineering disciplines beside the one in mathematical physics. It is pointed out that in all these applications the concept of pendulum is used as a vehicle of knowledge and mainly on the basis of structural

analogy. Finally, its role in an example of the consciousness-manifesting phenomenon will be discussed.

#### 19 SOME BASIC ASPECTS OF COMPUTER TECHNOLOGY IN VEDIC LITERATURE

#### Vinod Kumar

Department of Computer Science, GKV, Hardwar

Vedas are considered to be the first written book of the world and a treasure of knowledge containing all kinds of knowledge in the world in its basic form. Computer technology consists of hardware, software and humanware. Computer hardware basically depends upon the controlled flow of electricity keeping its level very low (-12V to 12V). Several mantras of Rigveda mentions about the advantage of controlled flow of electricity that provides the basis for hardware design of computers. Mantras, concerning the use of air and electricity for massage transmission, are also available in Vedic literature. Also, the Sutras of Vedic mathematics like "Ekadhiken Poorvena" have tremendous power of algorithmic representation.

The present paper aims at exploring the roots of electron flow, microwave transmission, computer networking and algorithmic representation of solution process in the Vedic literature.

#### 20 HISTORY OF DEVELOPMENT OF BINOMIAL EXPANSIONS

#### V. K. Sharma and Anita Sharma

Dep. of Mathematics and Statistics, GKV, Hardwar

In this paper contribution of various mathematicians like Varahmihir, Mahaviracharya, Bhaskaracharya, Umar Khayyam, Pascal, Newton and Gauss etc. in development of Binomial expansion and its validity is discussed.

#### 21 A DEVELOPEMENT OF APPROXIMATION THEORY IN INDIA — A CRITICAL REVIEW

#### S. P. Singh

Dep. of Mathematics, Guru Ghasi Das University, Bilaspur (Chhattisgarh)

## 22 FIXED POINTS FOR SET-VALUED MAPS

Devanand Ram, P. P. Murty and S. N. Lal

Dep. of Mathematics, Faculty of Science, BHU, Varanasi.

It is well known that Banach Contraction principle has been extended and generalised in several different forms by various authors. In this paper we have established a common fixed point theorem considering the Greguš  $\Phi$ -contraction in the setting of set-valued mappings.

### 23 ALGEBRA FOR COMPUTERS

#### P. Pradhan and Girish Sharma

Dept. of Mathematics, GKV, Hardwar Dep. of Computer Science, Hindu P.G. College, Sonipat (Haryana)

This paper is presenting the truth & validity of the prime (simple) & compound statement using connectives and attempts are made to establish the rule of modern algebra by implementing in terms of Boolean Algebra. Switching circuits are also discussed along with some digital circuits as an application of Boolean Algebra.

## 24 FIXED POINT THEOREMS FOR MEIR-KEELER TYPE HYBRID CONTRACTION

#### S. L. Singh and Anita Tomar

Dep. of Mathematics, GKV, Hardwar

Beg and Azam initiated the study of fixed points for a pair of single-valued and multivalued maps satisfying Meir-Keeler type contraction condition. The intent of this paper is to obtain fixed point theorems for a Meir-Keeler type hybrid contractions, which extend and improve main results.

## 25 TRANSFORMATION OF GEOMETRICAL FIGURES

#### Nidhi Handa

Dep. of Mathematics & Statistics, KGM, GKV, Hardwar

This paper aims at the conversion of area i.e. the conversion from rectangle into square and vice-versa, etc. It focuses on how the four-sided complicated structures have been simplified in the sulb-sutra. Here the endeavour has been also made to highlight the method of construction and proof of each sutra.

#### 26 USEFUL WEB LINKS ON HISTORY OF MATHEMATICAL SCIENCES

#### Man Mohan

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The Internet is a wonderful place to explore the vast amount of information that it makes available around the world, but without borders and often confusing the explorer. Even well prepared surfers stumble aimlessly through cyberspace using

hit-or-miss methods in search of useful information with few results, little substance and lots of frustration. The purpose of this article is to encourage and facilitate the teaching and learning of history of mathematics at all levels, using the Web. Some of the most useful resources on the history of mathematics are identified, described, and hyperlinked. The resources are divided into twelve sections. While some websites may fall in more than one section, each website appears in what seemed to be the most relevant section.

## 27 EFFICIENT FAMILIES OF CHAIN ESTIMATORS FOR RATIO AND PRODUCT OF TWO POPULATION MEANS

#### H. S. Jhajj, M. K. Sharma and Lovleen Kumar Grover

Guru Nanak Dev University, Amritsar

Here we proposed efficient families of chain estimators for estimating ratio and product of two population means using known information on one of variable of the given two auxiliary variables. The bias and mean squared error of proposed families are obtained up to first approximation. The optimum mean square errors are also obtained. The gain in efficiency is illustrated empirically.

## 28 REDUNDANT SYSTEM WITH N-REPAIRABLE SERVERS

#### Charanjeet Singh and Madhu Jain

Dep. of Mathematics, Guru Nanak Dev University, Amritsar Dep. of Mathematics, Institute of Basic Sciences Khandari, Agra

This investigation deals with steady state availability analysis with r repairmen where  $k_i$ - $n_i$ - out system fails, if  $n_i$ - $k_i$  units fail in i<sup>th</sup> server. The Hessenberg matrix determinants approach in recursive manner along with Markov chain balance equations has been provided to determine the availability analysis. The parallel series replicated technique has been adopted for illustration of steady state availability.

## 29 ROLE OF DYNIMICAL SYSTEMS AND ITS IMPLICATIONS

#### R. P. Pant and K. Jha

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The basic goal of the theory of dynamical systems is to understand the eventual or asymptotic behaviour of an iterative process. An important notion in the study of the dynamical systems is the stability or persistence of the system under small changes or perturbations, which is the concept of structural stability and the concept of structural stability is extremely important in applications of the theory of the dynamical systems. Dynamical systems occur in all branches of science, from the differential equations of classical mechanics in physics to the difference equations of mathematical economics and biology. The last thirty-five years have seen an explosion of interest in the study of the non-linear dynamical systems. Scientists in all disciplines have come to realize the power and the beauty of the geometric and qualitative techniques developed in the dynamical systems. The aim of the present paper is to give a brief account on the history, important results and some applications of dynamical systems.

## 30 **QUASI-CONTRACTIONS AND APPROXIMATE FIXED POINTS**

#### S. L. Singh and Bhagwati Prasad Chamola

Department of Mathematics, GKV, Hardwar

Approximate fixed points of the Banach contraction on non complete metric spaces exist. In this note, we discuss approximate fixed points of quasi-contractions on metric spaces.

## 21 PERFORMANCE EVALUATION OF COMPUTER COMUNICATION SYSTEMS

#### Manisha Sharma, P. K. Yadav and Avanish Kumar

Bansal Institute of Engg. & Tech. Bhopal, C.B.R.I., Roorkee Bundelkhand University, Jhansi

A Computer Communication System consists of a set of processing nodes, interconnected by to some extent by communication links. In a non-redundant system, each processing node contains exactly one processor and there is at most one link between any pair of processing nodes. The present paper discusses a general queue model to the system represent transmitter and receiver connected through communication channel. The mean response time, based on mean waiting time, mean transmitting time and mean service time is than obtained to determine the performance of the systems.

#### 32 OPERATION RESEARCH - A HISTORICAL NOTE

#### M. P. Singh, P. K. Yadav and Avanish Kumar

G.K.V. Hardwar, C.B.R.I., Roorkee, Bundelkhand University, Jhansi

The Operation Research is one of the important disciplines of the Mathematical Sciences, which deals a large number of problems related to Management, Engineering, Industry, Bio-Sciences etc. The role of these techniques is involving the day-to-day matters of the human beings. It suggests how to best use of the resources in lucid manner. The present paper focuses on the various stages of the development of the Operation Research and its concern ones.

## 33 HISTORY AND DEVLOPMENT OF BANACH CONTRACTION PRINCIPLE

#### U. C. Gairola

Dep. of Mathematics, Pauri Campus of HNB Garhwal University, Pauri

Using the concept of convergence of a cauchy sequence in a normed linear space S. Banach gave an important, concept of a special space which is named as BANACH spaces later on. This space plays a vital role in applicable Mathematics. The most and the first important application of Banach concept is used by himself by introducing contraction principle in metric spaces. In this paper I shall discuss the history, development and some basic applications of Banach contraction principle.

#### 34 PERMUTATION AND COMBINATION WITH ITS BRIEF HISTORY IN ANCIENT MATHEMATICS

#### Naubahar Singh

VMIC Sector-1, BHEL, Hardwar

In this paper an attempt is made to sketch a brief account of its history and discuss a few glimpses of Bhaskaracharya's II, techniques in treating the problems concerning to permutation and combination.

## 35 STABILITY OF MANN AND ISHIKAWA ITERATIVE PROCEDURES FOR CONTRACTIVE MAPS

#### S. L. Singh and Charu Bhatnagar

Department of Mathematics, GKV, Hardwar

Let X be a Banach spaces and T a self map of X. Let  $x_0$  be a point of X and  $x_{n+1} = Tx_n$  denotes an iteration procedure which yields a sequence of points  $\{x_n\}$ . Suppose that  $\{x_n\}$  converges to a fixed point p of T. Let  $\{y_n\}$  denote an arbitrary sequence in X and  $\epsilon_n = \|y_{n+1} - Ty_n\|$ . If  $\lim n \epsilon_n = 0$  implies that  $\lim_n y_n = p$  then the iteration procedure is to be T-stable. Let T:X->X, then there exists a constant satisfying  $0 \le c < 1$  such that, for each  $x,y \le X$ ,  $\|Tx-Ty\| \le cM(x,y)$  Where,  $M(x,y) = \max[\|x-y\|, \|x-Tx\|, \|y-Ty\|, 1/2\{\|x-Ty\| + \|y-Tx\|\}]$  In this paper, we show that several interval iteration procedures are stable for maps T satisfying above condition.

#### 36 SOME FIXED POINT THEOREMS ON 2-METRIC SPACES

#### D. D. Sharma

Department of Mathematics, GKV, Hardwar.

This paper establishes some fixed point theorem for Matkowski-type contraction mappings defined on 2-metric spaces.

## ANALYSIS OF STRESS- INTENSITY FACTOR FOR AN EPITROCHOIDALDISC UNDER TWO CONCENTRATED FORCES

#### Rasid Ali, M. P. Singh and Parul Rani

Dep. of Mathematics, Vishveswarya Institute of Engg. & Tech. Dadri, G.B.Nagar Dep. of Mathematics and Statistics, G KV, Hardwar

Complex variable technique has been found to be very effective for solving plane, axisymmetric and boundary value problems in Solid Mechanics. Employing this

method, the first fundamental plane problems of an epitrochoidal disc has been discussed. It is assumed that the plate is thin, isotropic and homogeneous under two concentrated forces applied at the extremities of its major axis of the exterior bounded epitrochoid. Stresses are found out and the variation of stress-intensity factor has been studied graphically. The solution of an elastic circular plate and that of a plate in the form of a cardioid has been obtained as particular cases with same boundary conditions.

## 38 MAGNETIC TORQUE EFFECT ON SATELLITE MOTION

#### Parsan Kaur and Rashmi Bhardwaj

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Magnetic disturbance torque results from the interaction between the spacecraft's residual magnetic field and the geomagnetic field. The primary sources of magnetic disturbance torques are (i) spacecraft magnetic moments, (ii) eddy currents and (iii) hysteresis. Of these, the spacecraft's magnetic moment is usually the dominant source of disturbance torques. Let us consider a rigid satellite S moving around the Central Body C. We further suppose that the orbital plane of satellite coincides with the equatorial plane of the central body. We discussed that the non-linear rotational equations of motion of the planar oscillation of a satellite in an elliptic orbit under the influence of magnetic torque are non-integrable. Also we have observed graphically that in the Earth-Moon system, the Melnikov's function has a simple zero and hence the equations of motions are non integrable.

#### 39 AN APPROCH TO NEW TRICORNS

#### S. L. Singh And Ashish Negi

G.K.V. Hardwar, Govind Ballabh Pant Engg. College, Pauri Garhwal

Recently using Mann iterative procedure, Kumar [3] (see also Rani [4]) have introduced superior Julia sets for polynomials. In this paper following the same techniques we obtain new tricorns arising from quadratic polynomials.

#### 40

## EFFECT OF DAMPING ON VIBRATIONS OF AN ISOTROPIC NON-HOMOGENEOUS CIRCULAR PLATE OF PARABOLICALLY VARYING THICKNESS

#### Rajendra Kumar, Virendra Arora and Manu Gupta

Dep. of Mathematics, J. V. Jain College, Saharanpur Dep. of Mathematics and Statistics, GKV, Hardwar

The research in the field of vibration is unceasingly acquiring immense importance in modern science as it has in every field of applied science today. The effect of vibration on design of machine, multistoried building, radio-telescope, nuclear reactor technology, earthquake resistant structure and various other engineering structures is of immense importance. Today we are going ahead in space technology where problems of plates continuously supported by elastic and visco-elastic media are considered. Also effect of damping is applied for practical use.

This paper deals with the free vibrations of damped non-homogeneous isotropic elastic circular plate of parabolically varying thickness. The modulus of elasticity 'E' and density ' $\rho$ ' of the plate material are assumed to vary linearly with the radius vector. The transverse displacement is expressed as a power series in terms of the radial co-ordinates and Frobenius method is used to solve the governing differential equation of motion. Frequency equations have been derived for the two combinations of boundary condition namely clamed –simply supported and clamped plate. The numerical results corresponding to the first two modes of vibrations have been computed for different values of taper constant, non-homogeneity and damping parameter.

#### 41

#### A STUDY OF ANCIENT ASTRONOMICAL INSTRUMENTS AND THEIR RELEVANCE TO MODERN TIME

#### V. K. Sharma and Mustafizur Rahman

Dep. of Mathematics and Statistics, GKV, Hardwar

This paper gives a concise account of ancient and modern astronomical instrument with the development of astronomical science in India. On the other

hand this account has been placed in the context of historical developments of instrument in India. In addition to ancient instrument the brief account of contributions by *Brahmagupta* and Raja Jai Singh is also included.

#### 42 NUMBERS IN VEDIC LITERATURE

#### Savitri Verma and Shiva Mittal

Dep. of Mathematics, Dr. S. P. M. Govt. Degree College, Allahabad

This paper intends to survey theory of numbers in Vedic literature.

## 43 A DEVELOPMENT OF THE SCIENCE OF COMBINATORICS IN ANCIENT INDIA

#### Parmeshwar Jha and N. K. Sinha

Saraswati Sadan and Vidyapuri, Supaul (Bihar) Dep. of Mathematics, Purner College, Purnen (Bihar)

Importance of the topic 'combinatories' in mathematics cannot be over emphasised. We have become so familiar with the subject that we seldom feel necessary to think how and when it was invented. Consequently very little efforts have been made in the past to record its gradual evolution especially in ancient India. The present paper is an attempt in this direction. There are grounds to show that its origin can be traced back to the period of Vedas. Principles and techniques for the solution of various types of combinatorial problems are also found to exist in the Sushrutn Samhita, Pingala's Chandah Sutra, Jaina Canonical works and other literature of ancient India.Moreever, Inter Indian Scholars, viz., Mahavirracharya, Halayudha, Bhaskaracharyn, Narayana Pandita and others have also explained several techniques in their works. On the whole, remarkable progress in the field has been made in different periods. The paper discusses all these achievement in detail and presents a chronological account of the growth and development of the science of combinatorics in ancient India.

#### 44

## MHD UNSTEADY FLOW OF A DUSTY VISCO-ELASTIC FLUID INDUCED BY THE MOTION OF A SEMI-INFINITE PLATE MOVING WITH VELOCITY DECREASING EXPONENTIALLY WITH TIME

#### B. S. Rawat

Department of Mathematics, HNB Garhwal University, Srinagar

In this paper, we have considered the velocities of dust particle and dusty fluid are obtained in presence of magnetic fields for the flow induced in an incompressible dusty fluid by the motion of semi-infinite flat plate moving with velocity decreasing exponentially with time. The velocity distribution of dusty fluid, dust particle and distribution of magnetic field in case of dusty fluid are represented graphically. Effect of elastic parameter, Hartmann number on the velocity and magnetic profile are also discussed.

#### 45 A COMPARISON OF BANACH AND KANNAN FIXED POINT THEOREMS

S. L. Singh, Anurag Naithani, R.P. Pant

Dep. of Mathematics & Statistics, GKV, Hardwar.

#### 46 COMMON FIXED POINT THEOREMS FOR FAMILY OF CONTRACTIVE TYPE MAPPINGS

#### S. L. Singh and Ashish Kumar

Dep. of Mathematics and Statistics, GKV, Hardwar

Common fixed point theorems have been proved for a family of mappings satisfying a minimal type commutative and contractive conditions. A fixed point theorem using E.A. property is also obtained.

## 47 ON COMMON FIXED POINT THEOREMS IN D-METRIC SPACES

#### Virendra Arora and Mohini Pundir

Dep. of Mathematics and Statistics, GKV, Hardwar

In this paper, fixed point theorems for a weakly compatible mappings in D-metric spaces satisfying certain strict contractive conditions are proved which include the fixed point theorems of Dhage as the special cases under weaker conditions.

### 48 NEW APPROACH TO GRAPHICAL MODELING

#### S. L. Singh and Manish Kumar

Dep. of Mathematics & Statistics, GKV, Hardwar, Dep. of Comp. Sc. & Engg., SRM CEM, Faizabad Road, Lucknow

Fractal Theory is a newly developed branch of Computer Graphics, based on feedback system. Images can be generated not having definite structures. Fractals are everywhere, e.g., medical science, wavelet theory, telecommunications, image compression and several other fields. This technique is simple to understand and easy to compute to solve problems, which yields realistic models.

## 49 DIFFERENT TYPES OF FUZZY MEASURES AND ITS APPLICATIONS IN DECISION THEORY

#### R. C. Dimri

Dep. of Mathematics, H.N.B. Garhwal University, Srinagar

Fuzzy set theory has formulated different types of uncertainty, which were before either not recognized properly or considered under probability theory. Various special classes of measures can be generated by the concept of fuzzy measure. Fuzziness can be introduced at several points in the presently existing models of

decision making. This contribution investigates some class of fuzzy measures and relationship among them.

#### 50 AN IDENTITY INVOLVING GOLDEN RATIO -A HISTORICAL PERSPECTIVE

#### Jamuna Prasad Ambasht

34 Westpine Court, Columbia, S.C. 29212 (U.S.A.)

The purpose of this talk is to make a modern mathematician travel through the lanes of Channai and acquaint oneself with the days of Ramanujan's survival amidst all hardships. The case in point is the nested square-roots, as also the continued fractions.

If one can recall the letters from an Indian Clerk to an English Professor, one could not afford to miss that part wherein Ramanujan claims Beta and Gamma functions involving integration lead to elegant identities. Hardy had no difficulty in understanding this. What threw him off was another assertion using the fifth roots of x in the numerators and the denominators of continued fractions.

The claim we are making is that a golden ratio g satisfies  $g^2 = g+1$  and therefore turns out to be u and v of Ramanujan and therefore fitsinto his identity involving powers of u and v. (see last equation of Ramanujan's letter to Hardy). This leads to beautiful new results in case of nested square-roots and also continued fractions as we can always express g in these forms.

#### . 51 MATHEMATICAL ENGLISH

#### Asim Majumdar and Sandeep Bhakat

Siksha-Satra, Visva Bharati, Sriniketan

In our country where different provincial languages are spoken English plays a very significant role as a means of communication. The objectives of the project are to teach English language to the first generation learners particularly for their English literacy by developing a new type of method for English language

rearning. Some mathematical tools and new concepts can justify the methodology. The project also aims at developing research-based techniques for easy and scientific English learning. The project also undertakes to prepare a wordbook designed with the words widely used in learning of English language in its basic level. The project attempts at characterizing the words in terms of usage in English language. Whether the reading materials developed in this project are very effective and scientific can be verified with the help of mathematical sciences particularly by the application of Fuzzy set theory to evaluate the potentiality of a word of different order.

As for example: Consider the set of all English words consisting of 3 letters with 'N' in the first position. There are 26x26 numbers of such combinations out of which 17 are considered English words. The problem of evaluating the potential value of one such word with respect to its frequency of use in communicating English. This potential value can be derived by the application of fuzzy set theory.

## 52 AIR POLLUTION AND ITS EFFECTS ON ATMOPSPHEREIC PROPERTIES

#### Virendra Arora and Ajendra Kumar

Dep. of Mathematics and Statistics, GKV, Hardwar.

Air pollution has posed a serious problem for our atmospheric properties. In this paper air pollution and its effects on human being, vegetation, materials etc. has been discussed.

## 53 EVOLUTION AND DEVELOPMENT OF NUMBER SYSTEM

#### N. A. Pande

Dep. of Mathematics, NES College, Nanded (Maharastra)

## 54 APPROXIMATE FIXED POINT THEOREMS

#### Abha Singh

Bela, Via Narayanpur, Mirzapur(U. P.)

### 55 WEAKLY TIGHT FUNCTIONS AND THEIR DECOMPOSITION

#### Pramila Srivastava, Mona Khare and Bhawna Singh

Allahabad Mathematical Society, 10, C.S.P. Singh Marg, Allahabad, India Department of Mathematics, University of Allahabad, Allahabad

The present paper deals with the study of a weakly tight function and its relation to tight and cotight functions. We have proved that every tight function is weakly tight and also every cotight function is weakly tight. We have obtained that a Jordan decomposition type theorem for a locally bounded function which is weakly tight followed by the notion of total variation. In addition, it is proved that if  $\beta$  is additive and modular, then the decomposed parts  $\beta$ + and  $\beta$ - are super additive and super modular.

#### 56 DOUBLE INDUCTION

C. S. Bisht

Department of Mathematics, KumaunUniversity, Nainital

#### 57 CONTRIBUTION OF SOME PROMINENT ANCIENT MATHEMATICIANS

D. L. Despande

Dep. of Mathematics, NES College, Nanded (Maharastra)

## 58 RULES IN VEDIC MATHEMATICS AND PROGRAMMING

#### S. L. Singh and Nilam Aswal

Dep. of Mathematics and Statistics, GKV, Hardwar

Computer programming based on certain rules of Vedic mathematics are discussed.

#### 59 PICARD ITERATIVE PROCEDURE IN ĀRYABHAŢIYAM

#### P. Pradhan

Dep. of Mathematics and Statistics, GKV, Hardwar

Iterative procedures akin to Picard iterations are used by Āryabhaṭa (b. 499 A.D.) and later mathematicians of Vedic traditions. This paper intends to discuss this procedure and related problems.

## 60 ARYABHATT – HIS CONTRIBUTIONS TO MATHEMATICS

#### D. S. Hooda

Kurukshetra University, Kurukshetra

## 61 APPLICATION OF LIAPUMOR FUNCTION TECHNIQUE IN THE STUDY OF ENTRY PROFILE OF A SPACE VEHICLE

#### B. Ram

Dep. of Mathematics, Tehri Campus of HNB Garhwal University, Tehri Garhwal

### 62 APPLICABLE MATHEMAICS: AN OVERVIEW

#### V. K. Katiyar

Dep. of Mathematics, IIT, Roorkee

Due to increasing demand of mathematical techniques in developing analytical and experimental models in all branches of science and technology for industrial development of the country, an overview will be presented as an application of mathematics in Biological, Social, Economic and in industrial systems.

### 63 A DEVLOPMENT OF HYBRID CONTRACTIONS AND COINCIDENCES

#### S. L. Singh and Amal M. Hashim

Gurukula Kangri Vishwavidyalaya, Hardwar

Study of hybrid contractions on metric spaces was initiated by S. L. Singh, Chitra Kulshrestha, R. Bhaskran, P. V. Subrahmanyam and Olga Hadzic during 1980-1984. Thereafter a number of papers have appeared in this line. The purpose of this talk is to present a brief historical development of hybrid contractions, and we also present some new coincident theorems for very general type of hybrid contractions have E.A. property.

#### 64 संस्कृत वाङ्मय एवं अंक विद्या

#### जयकृष्ण गोदियाल

हे0 नं0 ब0 गढ़वाल विश्वविद्यालय, पौड़ी परिसर पौडी 246001 उत्तरांचल

संस्कृत वाङ्मय संसार में सर्वाधिक प्राचीन एवं उपयोगी है। इस भाषा के साहित्य में विषयों की विविधता इसकी विशेषता है। साहित्यिक विषयों के साथ-साथ वैज्ञानिक विषयों के प्रतिपादन की इसकी प्रवृति के कारण ही सम्पूर्ण विश्व जनमानस अत्याधुनिक इस युग में भी संस्कृत वाङ्मय के अध्ययन की ओर गहराई से उन्मुख है। वैदिक वाङ्मय वस्तुत: विज्ञान का भंडार है। संस्कृत वाङ्मय में यत्र-तत्र पड़े गणितीय विचारों को उद्घाटित करना ही इस प्रस्तौष्यअण शोध पत्र पर मुख्य विवेच्य विषय है।

#### 65 भारत में कूटविज्ञान

सुन्दर लाल

गणित विभाग डा० भीमराव अम्बेदकर विश्वविद्यालय, आगरा

भारत में कूटविज्ञान की प्राचीन परम्परा है। इस प्रपत्र में इन परम्पराओं का कूट लेखन एवं उसके प्रेषण के सम्बन्ध में प्रकाश डाला गया है।

### NATIONAL SEMINAR ON HISTORY OF MATHEMATICAL SCIENCES & APPLICABLE MATHEMATICS

#### Organised by

### DEPARTMENT OF MATHEMATICS AND STATISTICS GURUKULA KANGRI VISHWAVIDHYALAYA, HARDWAR

## DAILY PROGRAMME

#### MARCH 07, 2003

#### **VENUE: Conference Hall, Faculty of Management**

09.00	REGISTRATION				
10.00	Yajna by Prof. Bharat Bhushan & Acharya Prof. Ved Prakash, Pro VC				
10.30	INAUGURAL SESSION				
	Welcome Address by Prof. S. L. Singh				
	Organizing Secretary & Principal, Science College				
	Address by Prof. Mahavir Agrawal, Registrar				
	Address by Acharya Prof. Ved Prakash, Pro VC				
	Inaugural Address by Chief Guest Shri Ramphal Bansal				
	Senior Advocate, Delhi High Court				
	Address by Prof. B. S. Yadav				
	Chairman, Organizing Committee				
	Key Note Address by Prof. G. S. Pandey				
	Presidential Address by Prof. Swatantra Kumar, Vice-Chancellor				
	Vote of thanks by Prof. Virendra Arora				
	Organizing Secretary & Dean Faculty of Science				
11.30	TEA				

TEA

12.00 - 12.45	Yamataization Prof. B. S. Yadav, Delhi Chairperson: Prof. G. S. Pandey
12.45-13.15	Similarity of plane figure & geometric & group
	Dr. Vinod Mishra, SLIET, Longowal
	Chairperson: Dr. Kanhaiya Jha
13.15	LUNCH
15.00-15.30	Applicable Mathematics : An overview
	Prof. V. K. Katiyar, IIT, Roorkee
	Chairperson: Prof. S. P. Singh
15.30-16.00	Useful Web Links on History of Mathematical Sciences
	Dr. Man Mohan, Delhi
	Chairperson: A. B. Lohni
16.00-1630	Inauguration of Exhibition on Vedic Sciences followed by Tea
16.30-17.30	TALKS/PAPER READING SESSION
	Talks/papers by: Prof. J. K. Godiyal, Amal M. Hashim,
	Asim Majumdar, Sandeep Bhakat, N. A. Pandey, D. L. Despande, Nilam Aswal, P. Pradhan, B. Ram, B. S. Rawat, Ashish Kumar, Mohini Pundir
	Chairperson: Dr. Man Mohan
17.30-19.30	Visit to Har -ki- Pauri
19.45-20.30	DINNER

### **VENUE:** Faculty of Science

### March 08, 2003

09.30-10.00	Measure Theory-Historical Perspective
	Prof. Pramila Srivastava, Allahabad Mathematical Society, Allahabad
	Chairperson: Prof. B. S. Yadav
10.00-10.20	New Directions in Vedic Mathematics
	Prof. S. L. Singh, Hardwar
	Chairperson: Prof. J. M. C. Joshi

10.20-10.40	A Brief Survey of Ancient Mathematics			
	Dr. J. Achari, Nanded			
	Chariperson: Prof. Virendra Arora			
10.40-11.10	The Kriyakramakari Exposition on			
	Dr. Madhukar Mallayya, Trivandram			
	Chariperson: Dr. J. Achari			
11.10-11.25	TEA			
11.25- 11.45	Innovations in Kerala Astronomy			
	Dr. S. Madhavan, Trivandrum			
	Chairperson: Prof. R. S. Kaushal			
11.45-12.15	The Pendula in Mathematical Sciences			
	Prof. R. S. Kaushal, University of Delhi			
	Chairperson: Prof. Vinod Kumar			
12.15-12.40	Double Induction			
	Dr. C. S. Bisht, Kumaun University, Nainital			
	Chairperson : Dr. Madhukar Mallayya			
12.40-13.00	Fractals in Lilavati			
	Dr. Mamta Rani, Lucknow Chairperson: Dr. S. Madhavan			
12.00.14.15				
13.00-14.15	LUNCH			
14.15-17.00	Excursion to Rishikesh including a visit to Paramarth Niketan			
15.00-15.30	Executive Council Metting, ISHM			
17.00-19.00	Shri Ganga Aarti, Blessings by Svami Chidanand Sarsvati "Muni Ji" and Prasadam			
March 09, 2003				
19.30.10.00	Innovations in Kerala Astronomy			
	Dr. S. Madhavan, Trivandrum Chairperson: Prof. C. S. Bisht			
10.00-10.20	Algebra of Narayana			
	Dr. M.M. Dwivedi, Chitrakoot			
	Chairperson: Prof. M. P. Singh			

10.20-10.40	Relevance of History in Mathematics Teaching
	Prof. J. M. C. Joshi, Nainital
	Chairperson: Prof. Pramila Srivastava
10.40-11.00	A developement of Approximation Theory
	Prof. S. P. Singh, Guru Ghasi Das University, Bilaspur
	Chairperson: Dr. S. Madhavan
11.00-11.15	TEA
11.15-11.35	Mahavira;s Method of False Position
	Dr. Kanahiya Jha, Jamshedpur
	Chairperson: Prof. V. K. Sharma
11.35-11.55	The Pendula in Mathematical
	Prof. R. S. Kaushal, Delhi
•	Chairperson: Prof. G. S. Pandey
11.55-13.20	Talks/ paper by: Subhash Chander, B. B. Kulkarni, Ramesh Chand,
	Anita Sharma, Devanand Ram, P. Pradhan, Grish Sharma,
	Anita Tomar, Nidhi Handa, H. S. Jhajj, M. K. Sharma, L. K. Grover,
	Charanjeet Singh, K. Jha, B. P. Chamola, M. Sharma, P. K. Yadav,
	A. Kumar, U. C. Gairola, N. Singh, C. Bhatnagar, D. D. Sharma,
	P. Kaur, R. Bhardwaj, A. Negi, M. Gupta, M. Rahman, S. Verma,
	M. Kumar, R. C. Dimri.
•	Chairpersons: Prof. S. P. Singh and Prof. B. Ram
13.20-13.40	Valedictory Function
13.40	LUNCH

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S. A. Naimpally and B. D. Warrack: Proximity Space, Cambridge Univ. Press, U.K., 1970 (for books)

B. E. Rhoades: A comparison of various definitions of contractive mappings, Trans. Amer. Math. Soc. 226 (1977), 267-290 (For articles in journals, title of the articles are not essential in long review/survey articles.)

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